Solution to ECE Test #10 S07 #1

Given $\mathrm{X}(z)=\dfrac{(z-0.3)(z+0.2)}{z(z^2+0.9)}$, express $\mathrm{X}(z)$ in the partial fraction form

$$\mathrm{X}(z)=\frac{K_1}{z}+\frac{K_2z+K_3}{z^2+0.9}.$$

Find the numerical values of K_1 , K_2 and K_3 .
$K_1=-0.0667 \quad K_2=1.0667 \quad K_3=-0.1$

The inverse transform of $\mathrm{X}(z)$ can be expressed in the form

$$\mathrm{x}[n]=A_1\delta[n-1]+A_2a_1^n\sin(a_2n)\mathrm{u}[n]+A_3a_3^{n-1}\sin(a_4(n-1))\mathrm{u}[n-1].$$

Find the numerical values of A_1 , A_2 , A_3 , a_1 , a_2 , a_3 and a_4 .

$A_1=-0.0667 \quad A_2=1.1245 \quad A_3=-0.1054$
$a_1=0.9486 \quad a_2=\pi/2=1.507 \quad a_3=0.9486 \quad a_4=1.507$

$$\mathrm{X}(z)=\frac{(z-0.3)(z+0.2)}{z(z^2+0.9)}=-\frac{0.0667}{z}+\frac{K_2z+K_3}{z^2+0.9}$$

Multiply through by z and then let z approach infinity,

$$\lim_{z\to\infty} z\frac{(z-0.3)(z+0.2)}{z(z^2+0.9)}=\lim_{z\to\infty}\left(-z\frac{0.0667}{z}+\frac{K_2z^2+K_3z}{z^2+0.9}\right)$$

$$1=-0.0667+K_2 \Rightarrow K_2=1.0667$$

Let $z=1$ in $\dfrac{(z-0.3)(z+0.2)}{z(z^2+0.9)}=-\dfrac{0.0667}{z}+\dfrac{1.0667z+K_3}{z^2+0.9}$
and solve for B.

$$\frac{0.7\times1.2}{1.9}=-0.0667+\frac{1.0667+K_3}{1.9}\Rightarrow 0.84=-0.12667+1.0667+K_3$$
$$K_3=0.84+0.12667-1.0667=-0.1$$

$$\mathrm{X}(z)=-\frac{0.0667}{z}+\frac{1.0667z-0.1}{z^2+0.9}$$

$$\mathrm{X}(z) = -\frac{0.0667}{z} + \frac{1.0667}{0.9486}\frac{0.9486z}{z^2 + 0.9} - \frac{0.1}{0.9486}\frac{0.9486}{z^2 + 0.9}$$

$$\mathrm{X}(z) = -\frac{0.0667}{z} + 1.1245\frac{0.9486z}{z^2 + 0.9} - 0.1054\frac{0.9486}{z^2 + 0.9}$$

$$\mathrm{x}[n] = \begin{bmatrix} -0.0667\delta[n-1] + 1.1245(0.9486)^n \sin(\pi n / 2)\mathrm{u}[n] \\ -0.1054(0.9486)^{n-1} \sin(\pi(n-1)/2)\mathrm{u}[n-1] \end{bmatrix}$$

Solution to ECE Test #10 S07 #2

Given $\mathrm{X}(z) = \dfrac{(z-0.3)(z+0.2)}{z(z^2+0.8)}$, express $\mathrm{X}(z)$ in the partial fraction form

$$\mathrm{X}(z) = \frac{K_1}{z} + \frac{K_2 z + K_3}{z^2+0.8}.$$

Find the numerical values of K_1 , K_2 and K_3.
$K_1 = -0.075 \quad K_2 = 1.075 \quad K_3 = -0.1$

The inverse transform of $\mathrm{X}(z)$ can be expressed in the form

$$\mathrm{x}[n] = A_1\delta[n-1] + A_2 a_1^n \sin(a_2 n)\mathrm{u}[n] + A_3 a_3^{n-1}\sin(a_4(n-1))\mathrm{u}[n-1].$$

Find the numerical values of A_1 , A_2 , A_3 , a_1 , a_2 , a_3 and a_4.

$A_1 = -0.075 \quad A_2 = 1.1202 \quad A_3 = -0.1118$
$a_1 = 0.8944 \quad a_2 = \pi/2 = 1.507 \quad a_3 = 0.8944 \quad a_4 = 1.507$

$$\mathrm{X}(z) = \frac{(z-0.3)(z+0.2)}{z(z^2+0.8)} = -\frac{0.075}{z} + \frac{K_2 z + K_3}{z^2+0.8}$$

Multiply through by z and then let z approach infinity,

$$\lim_{z\to\infty} z\frac{(z-0.3)(z+0.2)}{z(z^2+0.8)} = \lim_{z\to\infty}\left(-z\frac{0.075}{z} + \frac{K_2 z^2 + K_3 z}{z^2+0.8}\right)$$

$$1 = -0.075 + K_2 \Rightarrow K_2 = 1.075$$

Let $z = 1$ in $\dfrac{(z-0.3)(z+0.2)}{z(z^2+0.8)} = -\dfrac{0.075}{z} + \dfrac{1.075z + K_3}{z^2+0.8}$
and solve for B.

$$\frac{0.7\times 1.2}{1.8} = -0.075 + \frac{1.075 + K_3}{1.8} \Rightarrow 0.84 = -0.135 + 1.075 + K_3$$
$$K_3 = 0.84 + 0.135 - 1.075 = -0.1$$

$$X(z) = -\frac{0.075}{z} + \frac{1.075z - 0.1}{z^2 + 0.8}$$

$$X(z) = -\frac{0.075}{z} + \frac{1.075}{0.8944}\frac{0.8944z}{z^2 + 0.8} - \frac{0.1}{0.8944}\frac{0.8944}{z^2 + 0.8}$$

$$X(z) = -\frac{0.075}{z} + 1.202\frac{0.8944z}{z^2 + 0.8} - 0.1118\frac{0.8944}{z^2 + 0.8}$$

$$x[n] = \begin{bmatrix} -0.075\delta[n-1] + 1.202(0.8944)^n \sin(\pi n / 2)u[n] \\ -0.1118(0.8944)^{n-1} \sin(\pi(n-1)/2)u[n-1] \end{bmatrix}$$

Solution to ECE Test #10 S07 #3

Given $\mathrm{X}(z)=\dfrac{(z-0.3)(z+0.2)}{z(z^2+0.7)}$, express $\mathrm{X}(z)$ in the partial fraction form

$$\mathrm{X}(z)=\frac{K_1}{z}+\frac{K_2 z+K_3}{z^2+0.7}.$$

Find the numerical values of K_1 , K_2 and K_3 .
$K_1=-0.086 \quad K_2=1.086 \quad K_3=-0.1$

The inverse transform of $\mathrm{X}(z)$ can be expressed in the form

$$\mathrm{x}[n]=A_1\delta[n-1]+A_2 a_1^n \sin(a_2 n)\mathrm{u}[n]+A_3 a_3^{n-1}\sin(a_4(n-1))\mathrm{u}[n-1].$$

Find the numerical values of A_1 , A_2 , A_3 , a_1 , a_2 , a_3 and a_4 .

$A_1=-0.086 \quad A_2=1.298 \quad A_3=-0.1195$
$a_1=0.8367 \quad a_2=\pi/2=1.507 \quad a_3=0.8367 \quad a_4=1.507$

$$\mathrm{X}(z)=\frac{(z-0.3)(z+0.2)}{z(z^2+0.7)}=-\frac{0.086}{z}+\frac{K_2 z+K_3}{z^2+0.7}$$

Multiply through by z and then let z approach infinity,

$$\lim_{z\to\infty} z\frac{(z-0.3)(z+0.2)}{z(z^2+0.7)}=\lim_{z\to\infty}\left(-z\frac{0.086}{z}+\frac{K_2 z^2+K_3 z}{z^2+0.7}\right)$$

$$1=-0.086+K_2 \Rightarrow K_2=1.086$$

Let $z=1$ in $\dfrac{(z-0.3)(z+0.2)}{z(z^2+0.7)}=-\dfrac{0.086}{z}+\dfrac{1.086z+K_3}{z^2+0.7}$
and solve for B.

$$\frac{0.7\times 1.2}{1.7}=-0.086+\frac{1.086+K_3}{1.7}\Rightarrow 0.84=-0.1462+1.086+K_3$$
$$K_3=0.84+0.1462-1.086=-0.1$$

$$\mathrm{X}(z) = -\frac{0.086}{z} + \frac{1.086z - 0.1}{z^2 + 0.7}$$

$$\mathrm{X}(z) = -\frac{0.086}{z} + \frac{1.086}{0.8367}\frac{0.8367z}{z^2 + 0.7} - \frac{0.1}{0.8367}\frac{0.8367}{z^2 + 0.7}$$

$$\mathrm{X}(z) = -\frac{0.086}{z} + 1.298\frac{0.8367z}{z^2 + 0.9} - 0.1195\frac{0.8367}{z^2 + 0.9}$$

$$\mathrm{x}[n] = \begin{bmatrix} -0.086\delta[n-1] + 1.298(0.8367)^n \sin(\pi n / 2)\mathrm{u}[n] \\ -0.1195(0.8367)^{n-1} \sin(\pi(n-1)/2)\mathrm{u}[n-1] \end{bmatrix}$$