Solution to ECE Test #9 S08 #1

1. $a(b^n)\sin(cn)u[n]\xleftarrow{\;z\;}\dfrac{0.5z}{z^2-0.8z+0.5}$

$$\alpha=\sqrt{0.5}=0.7071$$

$$2\alpha\cos(\Omega_0)=0.8\Rightarrow\cos(\Omega_0)=0.4/\alpha=0.5657\Rightarrow\Omega_0=0.9695$$

$$\frac{0.5z}{z^2-0.8z+0.5}=\frac{0.5}{\alpha\sin(\Omega_0)}\frac{\alpha\sin(\Omega_0)z}{z^2-0.8z+0.5}=0.8575\frac{0.5831z}{z^2-0.8z+0.5}$$

$$0.8575(0.7071)^n\sin(0.9695n)u[n]\xleftarrow{\;z\;}0.8575\frac{0.5831z}{z^2-0.8z+0.5}$$

2. (a) $x[n]\xleftarrow{\;z\;}\dfrac{1}{z+0.6}$ and $y[n]=\displaystyle\sum_{m=0}^{n}x[m]=\left[a+bc^{n-1}\right]u[n-1]$.

$$y[n]\xleftarrow{\;z\;}\frac{z}{z-1}\frac{1}{z+0.6}=\frac{5/8}{z-1}+\frac{3/8}{z+0.6}$$

$$\left[(5/8)+(3/8)(-0.6)^{n-1}\right]u[n-1]\xleftarrow{\;z\;}\frac{5/8}{z-1}+\frac{3/8}{z+0.6}$$

(b)   Find the numerical value of $x[8]$.

$$x[8]=\left[(5/8)+(3/8)(-0.6)^{8-1}\right]u[8-1]=\left[(5/8)+(3/8)(-0.6)^7\right]=0.6145$$

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1. $$a(b^n)\sin(cn)u[n] \xleftarrow{\ z\ } \frac{0.3z}{z^2 - 0.6z + 0.5}$$

$$\alpha = \sqrt{0.5} = 0.7071$$

$$2\alpha\cos(\Omega_0) = 0.6 \Rightarrow \cos(\Omega_0) = 0.6/\alpha = 0.4243 \Rightarrow \Omega_0 = 1.1326$$

$$\frac{0.3z}{z^2 - 0.6z + 0.5} = \frac{0.3}{\alpha\sin(\Omega_0)}\frac{\alpha\sin(\Omega_0)z}{z^2 - 0.6z + 0.5} = 0.4685\frac{0.6403z}{z^2 - 0.6z + 0.5}$$

$$0.4685(0.7071)^n\sin(1.1326n)u[n] \xleftarrow{\ z\ } 0.4685\frac{0.6403z}{z^2 - 0.6z + 0.5}$$

2. (a) $$x[n] \xleftarrow{\ z\ } \frac{1}{z - 0.5} \quad \text{and} \quad y[n] = \sum_{m=0}^{n} x[m] = \left[a + bc^{n-1}\right]u[n-1].$$

$$y[n] \xleftarrow{\ z\ } \frac{z}{z-1}\frac{1}{z-0.5} = \frac{2}{z-1} - \frac{1}{z-0.5}$$

$$\left[2 - (0.5)^{n-1}\right]u[n-1] \xleftarrow{\ z\ } \frac{2}{z-1} - \frac{1}{z-0.5}$$

(b) Find the numerical value of $x[8]$.

$$x[8] = \left[2 - (0.5)^{8-1}\right]u[8-1] = \left[2 - (0.5)^7\right] = 1.9922$$

Solution to ECE Test #9 S08 #1

1. $a(b^n)\sin(cn)u[n] \xleftarrow{\ Z\ } \dfrac{2.1z}{z^2 - 0.5z + 0.4}$

$$\alpha = \sqrt{0.4} = 0.6325$$

$$2\alpha\cos(\Omega_0) = 0.5 \Rightarrow \cos(\Omega_0) = 0.25 / \alpha = 0.3953 \Rightarrow \Omega_0 = 1.1644$$

$$\dfrac{2.1z}{z^2 - 0.5z + 0.4} = \dfrac{2.1}{\alpha\sin(\Omega_0)}\dfrac{\alpha\sin(\Omega_0)z}{z^2 - 0.5z + 0.4} = 3.6146\dfrac{0.581z}{z^2 - 0.5z + 0.4}$$

$$3.6146(0.6325)^n\sin(1.1644n)u[n] \xleftarrow{\ Z\ } 3.6146\dfrac{0.581z}{z^2 - 0.5z + 0.4}$$

2. (a) $x[n] \xleftarrow{\ Z\ } \dfrac{1}{z + 0.3}$ and $y[n] = \displaystyle\sum_{m=0}^{n} x[m] = \left[a + bc^{n-1}\right]u[n-1]$.

$$y[n] \xleftarrow{\ Z\ } \dfrac{z}{z-1}\dfrac{1}{z+0.3} = \dfrac{0.7692}{z-1} + \dfrac{0.2308}{z+0.3}$$

$$\left[0.7692 + 0.2308(-0.3)^{n-1}\right]u[n-1] \xleftarrow{\ Z\ } \dfrac{0.7692}{z-1} + \dfrac{0.2308}{z+0.3}$$

(b) Find the numerical value of $x[4]$.

$$x[4] = \left[0.7692 + 0.2308(-0.3)^{4-1}\right]u[4-1] = \left[0.7692 + 0.2308(-0.3)^3\right] = 0.7630$$