Solution of ECE 316 Test 4 Su08

1. Find the numerical values of the constants.

(a) $$10\left(-0.4\right)^{n}\sin\left(\pi n/8\right)\mathrm{u}\left[n\right]\overset{z}{\longleftrightarrow}\frac{b_2z^2+b_1z+b_0}{z^2+a_1z+a_0}$$

$$10\left(-0.4\right)^{n}\sin\left(\pi n/8\right)\mathrm{u}\left[n\right]\overset{z}{\longleftrightarrow}10\frac{-0.4\sin\left(\pi/8\right)z}{z^2-2\left(-0.4\right)\cos\left(\pi/8\right)z+\left(-0.4\right)^2}$$

$$10\left(-0.4\right)^{n}\sin\left(\pi n/8\right)\mathrm{u}\left[n\right]\overset{z}{\longleftrightarrow}\frac{-1.5308z}{z^2+0.7391z+0.16}$$

(b) $$Aa^{n}\sin\left(bn\right)\mathrm{u}\left[n\right]\overset{z}{\longleftrightarrow}\frac{12z}{z^2+0.64}$$

Using $$\alpha^{n}\sin\left(\Omega_0 n\right)\mathrm{u}\left[n\right]\overset{z}{\longleftrightarrow}\frac{\alpha\sin\left(\Omega_0\right)z}{z^2-2\alpha\cos\left(\Omega_0\right)z+\alpha^2}$$

$$\alpha=\sqrt{0.64}=0.8\ ,\ \Omega_0=\cos^{-1}\left(0\right)=\pi/2$$

$$15\left(0.8\right)^{n}\sin\left(\pi n/2\right)\mathrm{u}\left[n\right]\overset{z}{\longleftrightarrow}\frac{12z}{z^2+0.64}$$

2. Find the numerical values of the constants.

(a) $\left(Aa^n + Bb^n\right)\mathrm{u}\left[n\right] \overset{\mathrm{z}}{\longleftrightarrow} \dfrac{z\left(z-0.4\right)}{z^2+1.5z+0.3}$

$$\frac{z\left(z-0.4\right)}{z^2+1.5z+0.3} = z\left(\frac{1.622}{z+1.2623}+\frac{-0.6223}{z+0.2377}\right) = \frac{1.622z}{z+1.2623}-\frac{0.6223z}{z+0.2377}$$

$$\left(1.622\left(-1.263\right)^n - 0.6223\left(-0.2377\right)^n\right)\mathrm{u}\left[n\right] \overset{\mathrm{z}}{\longleftrightarrow} \frac{1.622z}{z+1.2623}-\frac{0.6223z}{z+0.2377}$$

(b) $A\,\mathrm{tri}\left(\left(n-n_0\right)/b\right) \overset{\mathrm{z}}{\longleftrightarrow} \dfrac{z^2+2z+1}{z^2}$

$$2\,\mathrm{tri}\left(\left(n-1\right)/2\right) = \delta\left[n\right]+2\delta\left[n-1\right]+\delta\left[n-2\right] \overset{\mathrm{z}}{\longleftrightarrow} \frac{z^2+2z+1}{z^2} = 1+2z^{-1}+z^{-2}$$

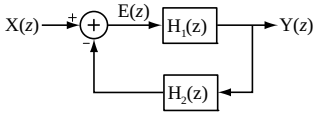
3. Determine whether these systems are stable or unstable (indicate by circling the correct answer) and explain how you know. (A correct answer without a correct explanation gets no points.)

(a) A system described by $2y[n]+3y[n-1]=x[n]$.

$$2Y(z)+3z^{-1}Y(z)=X(z)$$

$$H(z)=\frac{z}{2z+3} \Rightarrow \text{Pole at } 2z+3=0 \text{ or } z=-1.5 \Rightarrow \text{Unstable}$$

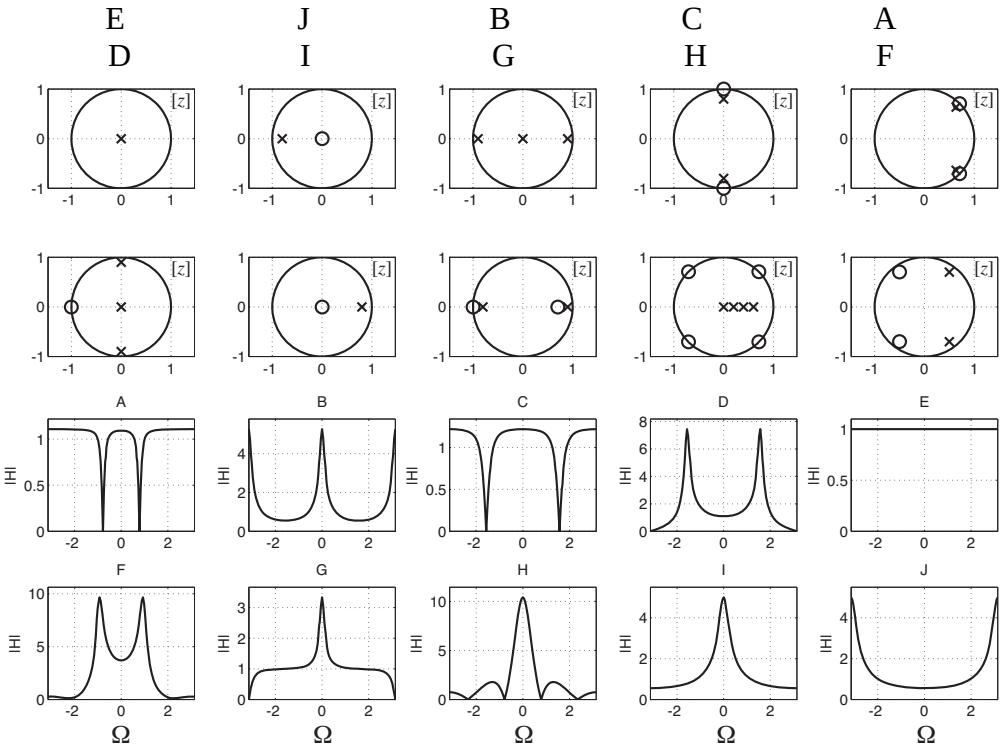
(b) A feedback system with $H_1(z)=\dfrac{0.7z}{z^2+0.6z+0.5}$ and $H_2(z)=z^{-1}$.



$$H(z)=\frac{\dfrac{0.7z}{z^2+0.6z+0.5}}{1+z^{-1}\dfrac{0.7z}{z^2+0.6z+0.5}}=\frac{0.7z}{z^2+0.6z+1.2}$$, Poles at $z=-0.3\pm j1.0536$

Unstable

4. Match the pole-zero diagrams to the magnitude frequency responses. (Write the corresponding letter above each pole-zero diagram.) (2 pts each)



Solution of ECE 316 Test 4 Su08

1. Find the numerical values of the constants.

(a) $$7\left(-0.5\right)^{n}\sin\left(\pi n/12\right)\mathrm{u}\left[n\right]\overset{\mathrm{z}}{\longleftrightarrow}\frac{b_2z^2+b_1z+b_0}{z^2+a_1z+a_0}$$

$$7\left(-0.5\right)^{n}\sin\left(\pi n/12\right)\mathrm{u}\left[n\right]\overset{\mathrm{z}}{\longleftrightarrow}7\frac{-0.5\sin\left(\pi/12\right)z}{z^2-2\left(-0.5\right)\cos\left(\pi/12\right)z+\left(-0.5\right)^2}$$

$$7\left(-0.5\right)^{n}\sin\left(\pi n/12\right)\mathrm{u}\left[n\right]\overset{\mathrm{z}}{\longleftrightarrow}\frac{-0.9059z}{z^2+0.9659z+0.25}$$

(b) $$Aa^{n}\sin\left(bn\right)\mathrm{u}\left[n\right]\overset{\mathrm{z}}{\longleftrightarrow}\frac{9z}{z^2+0.36}$$

Using $\alpha^{n}\sin\left(\Omega_0 n\right)\mathrm{u}\left[n\right]\overset{\mathrm{z}}{\longleftrightarrow}\frac{\alpha\sin\left(\Omega_0\right)z}{z^2-2\alpha\cos\left(\Omega_0\right)z+\alpha^2}$

$$\alpha=\sqrt{0.36}=0.6\ ,\ \Omega_0=\cos^{-1}\left(0\right)=\pi/2$$

$$15\left(0.6\right)^{n}\sin\left(\pi n/2\right)\mathrm{u}\left[n\right]\overset{\mathrm{z}}{\longleftrightarrow}\frac{9z}{z^2+0.36}$$

2. Find the numerical values of the constants.

(a) $\left(Aa^n + Bb^n\right)\mathrm{u}\left[n\right] \overset{z}{\longleftrightarrow} \dfrac{z\left(z-0.8\right)}{z^2+1.4z+0.2}$

$$\frac{z\left(z-0.8\right)}{z^2+1.4z+0.2} = z\left(\frac{1.893}{z+1.239}+\frac{-0.8927}{z+0.1615}\right) = \frac{1.893z}{z+1.239}-\frac{0.8927z}{z+0.1615}$$

$$\left(1.893\left(-1.239\right)^n - 0.8927\left(-0.1615\right)^n\right)\mathrm{u}\left[n\right] \overset{z}{\longleftrightarrow} \frac{1.893z}{z+1.239}-\frac{0.8927z}{z+0.1615}$$

(b) $A\,\mathrm{tri}\left(\left(n-n_0\right)/b\right) \overset{z}{\longleftrightarrow} \dfrac{z^2+2z+1}{z^2}$

$$2\,\mathrm{tri}\left(\left(n-1\right)/2\right) = \delta\left[n\right]+2\delta\left[n-1\right]+\delta\left[n-2\right] \overset{z}{\longleftrightarrow} \frac{z^2+2z+1}{z^2} = 1+2z^{-1}+z^{-2}$$

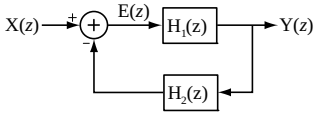
3. Determine whether these systems are stable or unstable (indicate by circling the correct answer) and explain how you know. (A correct answer without a correct explanation gets no points.)

(a) A system described by $3y[n] + 4y[n-1] = x[n]$.

$$3Y(z) + 4z^{-1}Y(z) = X(z)$$

$$H(z) = \frac{z}{3z+4} \Rightarrow \text{Pole at } 3z + 4 = 0 \text{ or } z = -1.333 \Rightarrow \text{Unstable}$$

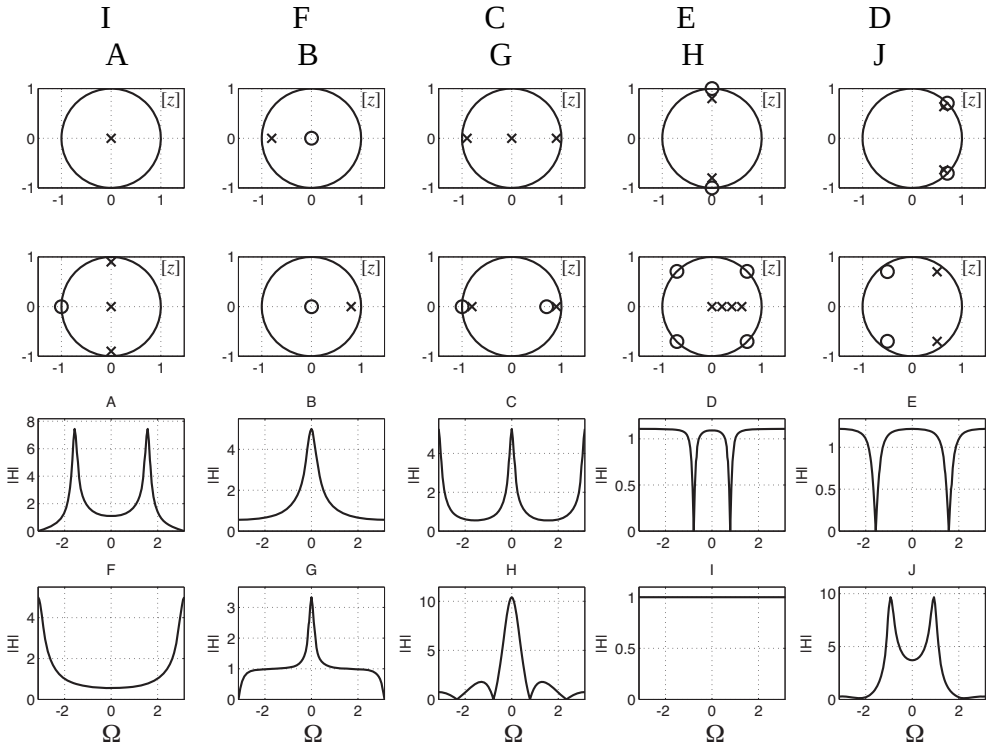
(b) A feedback system with $H_1(z) = \dfrac{0.4z}{z^2 + 0.6z + 0.5}$ and $H_2(z) = z^{-1}$.



$$H(z) = \frac{\dfrac{0.4z}{z^2 + 0.6z + 0.5}}{1 + z^{-1}\dfrac{0.4z}{z^2 + 0.6z + 0.5}} = \frac{0.4z}{z^2 + 0.6z + 0.9}$$, Poles at $z = -0.3 \pm j0.9$

Stable

4. Match the pole-zero diagrams to the magnitude frequency responses. (Write the corresponding letter above each pole-zero diagram.) (2 pts each)



Solution of ECE 316 Test 4 Su08

1. Find the numerical values of the constants.

(a) $$4\left(-0.3\right)^{n}\sin\left(\pi n/6\right)\mathrm{u}\left[n\right]\overset{z}{\longleftrightarrow}\frac{b_2z^2+b_1z+b_0}{z^2+a_1z+a_0}$$

$$4\left(-0.3\right)^{n}\sin\left(\pi n/6\right)\mathrm{u}\left[n\right]\overset{z}{\longleftrightarrow}4\frac{-0.3\sin\left(\pi/6\right)z}{z^2-2\left(-0.3\right)\cos\left(\pi/6\right)z+\left(-0.3\right)^2}$$

$$4\left(-0.3\right)^{n}\sin\left(\pi n/6\right)\mathrm{u}\left[n\right]\overset{z}{\longleftrightarrow}\frac{-0.6z}{z^2+0.5196z+0.09}$$

(b) $$Aa^{n}\sin\left(bn\right)\mathrm{u}\left[n\right]\overset{z}{\longleftrightarrow}\frac{18z}{z^2+0.49}$$

Using $$\alpha^{n}\sin\left(\Omega_0 n\right)\mathrm{u}\left[n\right]\overset{z}{\longleftrightarrow}\frac{\alpha\sin\left(\Omega_0\right)z}{z^2-2\alpha\cos\left(\Omega_0\right)z+\alpha^2}$$

$$\alpha=\sqrt{0.49}=0.7\ ,\ \Omega_0=\cos^{-1}\left(0\right)=\pi/2$$

$$25.714\left(0.7\right)^{n}\sin\left(\pi n/2\right)\mathrm{u}\left[n\right]\overset{z}{\longleftrightarrow}\frac{18z}{z^2+0.49}$$

2. Find the numerical values of the constants.

(a) $$\left(Aa^n + Bb^n\right)\mathrm{u}\left[n\right] \overset{z}{\longleftrightarrow} \frac{z\left(z+0.4\right)}{z^2+1.5z+0.2}$$

$$\frac{z\left(z+0.4\right)}{z^2+1.5z+0.2} = z\left(\frac{0.7907}{z+1.352}+\frac{0.2093}{z+0.1479}\right) = \frac{0.7907z}{z+1.352}+\frac{0.2093z}{z+0.1479}$$

$$\left(0.7907\left(-1.352\right)^n + 0.2093\left(-0.1479\right)^n\right)\mathrm{u}\left[n\right] \overset{z}{\longleftrightarrow} \frac{0.7907z}{z+1.352}+\frac{0.2093z}{z+0.1479}$$

(b) $$A\,\mathrm{tri}\left(\left(n-n_0\right)/b\right) \overset{z}{\longleftrightarrow} \frac{z^2+2z+1}{z^2}$$

$$2\,\mathrm{tri}\left(\left(n-1\right)/2\right) = \delta\left[n\right]+2\delta\left[n-1\right]+\delta\left[n-2\right] \overset{z}{\longleftrightarrow} \frac{z^2+2z+1}{z^2} = 1+2z^{-1}+z^{-2}$$

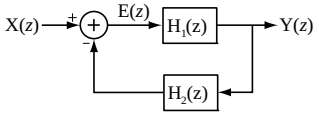
3. Determine whether these systems are stable or unstable (indicate by circling the correct answer) and explain how you know. (A correct answer without a correct explanation gets no points.)

(a) A system described by $5y[n] + 3y[n-1] = x[n]$.

$$5Y(z) + 3z^{-1}Y(z) = X(z)$$

$$H(z) = \frac{z}{5z+3} \Rightarrow \text{Pole at } 5z + 3 = 0 \text{ or } z = -0.6 \Rightarrow \text{Stable}$$

(b) A feedback system with $H_1(z) = \dfrac{0.2z}{z^2 + 0.6z + 0.5}$ and $H_2(z) = z^{-1}$.



$$H(z) = \frac{\dfrac{0.2z}{z^2 + 0.6z + 0.5}}{1 + z^{-1}\dfrac{0.2z}{z^2 + 0.6z + 0.5}} = \frac{0.2z}{z^2 + 0.6z + 0.7}$$, Poles at $z = -0.3 \pm j0.781$

Stable

4. Match the pole-zero diagrams to the magnitude frequency responses. (Write the corresponding letter above each pole-zero diagram.) (2 pts each)

E A J I D

F C H B G