Solution of EE 503 Test #1 S04

1. A CT signal is defined by $\mathrm{x}(t) = 10 + 3t - t^2$. Find the numerical values of

   (a) $\mathrm{x}(-2) = 0$

$$\mathrm{x}(-2) = 10 + 3(-2) - (-2)^2 = 10 - 6 - 4 = 0$$

   (b) $\mathrm{y}(3) = 11.25$ where $\mathrm{y}(t) = \mathrm{x}\left(\frac{t-1}{4}\right)$

$$\mathrm{y}(3) = \mathrm{x}\left(\frac{3-1}{4}\right) = \mathrm{x}\left(\frac{1}{2}\right) = 10 + 3\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^2 = 10 + \frac{3}{2} - \frac{1}{4} = 11\frac{1}{4} = 11.25$$

2. A DT signal, $\mathrm{x}[n]$, is periodic with period, 8. It is described over one period by

$$\mathrm{x}[n] = n(\mathrm{u}[n] - \mathrm{u}[n-5]) \ , \ 0 \le n < 8$$

Find the numerical value of $\mathrm{x}[1251] = 3$.

For any integer, n, $\mathrm{x}[1251] = \mathrm{x}[1251 + 8n]$. Let n be -156. Then

$$\mathrm{x}[1251] = \mathrm{x}[1251 + 8 \times (-156)] = \mathrm{x}[1251 - 1248] = \mathrm{x}[3]$$

$$\mathrm{x}[1251] = 3(\mathrm{u}[3] - \mathrm{u}[3-5]) = 3$$

3. A CT signal is defined by $\mathrm{x}(t) = \sin(2\pi t)\,\mathrm{rect}(5t)e^{-|t|}$. Find the numerical value of

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathrm{x}(t)\,dt = 0.$$

The integrand is an odd function and the limits are equal in magnitude and opposite in sign.

4. Find the numerical value of the signal energy of $\mathrm{x}[n] = -2\,\mathrm{tri}\left(\frac{n-3}{2}\right)$.

$$E_x = \sum_{n=-\infty}^{\infty} |\mathrm{x}[n]|^2 = \sum_{n=-\infty}^{\infty} \left|-2\,\mathrm{tri}\left(\frac{n-3}{2}\right)\right|^2 = 4\sum_{n=1}^{5} \mathrm{tri}^2\left(\frac{n-3}{2}\right) = 4\left[0^2 + \left(\frac{1}{2}\right)^2 + 1^2 + \left(\frac{1}{2}\right)^2 + 0^2\right] = 6$$

5. A DT system is described by $2\,\mathrm{y}[n] - 5\,\mathrm{y}[n-1] = \mathrm{x}[n]$ where x is the excitation and y is the response. Is it stable?

Characteristic Equation:

$$2\alpha - 5 = 0 \Rightarrow \alpha = \frac{5}{2} > 1$$

Unstable because the magnitude of at least one eigenvalue is greater than or equal to one.

6. A CT system is described by $y''(t) - 2y'(t) + y(t) = x(t) - x'(t)$ where x is the excitation and y is the response. Is it stable?

$$\lambda^2 - 2\lambda + 1 = 0 \Rightarrow (\lambda - 1)(\lambda - 1) \Rightarrow \lambda_{1,2} = 1,1$$

Unstable because the real part of at least one eigenvalue is greater than or equal to zero.

7. If $y[n] = x[n] * h[n]$ and $x[n] = u[n] - u[n-2]$ and $h[n] = 3\delta[n] + 2\delta[n+1]$ what is the numerical value of $y[1]$?

$$y[n] = (u[n] - u[n-2]) * (3\delta[n] + 2\delta[n+1])$$

$$y[n] = u[n] * 3\delta[n] - u[n-2] * 3\delta[n] + u[n] * 2\delta[n+1] - u[n-2] * 2\delta[n+1]$$

$$y[n] = 3u[n] - 3u[n-2] + 2u[n+1] - 2u[n-1]$$

$$y[1] = 3u[1] - 3u[1-2] + 2u[1+1] - 2u[1-1] = 3u[1] - 3u[-1] + 2u[2] - 2u[0] = 3 - 0 + 2 - 2 = 3$$

8. Let $x(t) = [\delta(t) - 4\delta(t-2)] * \text{comb}(t)$. Find the numerical value of its CTFS harmonic function, $X[k]$, at $k = 3$ using its fundamental period as the representation time.

$$x(t) = \delta(t) * \text{comb}(t) - 4\delta(t-2) * \text{comb}(t) = \text{comb}(t) - 4\underbrace{\text{comb}(t-2)}_{=\text{comb}(t)}$$

$$x(t) = \text{comb}(t) - 4\,\text{comb}(t) = -3\text{comb}(t) \Rightarrow X[k] = -3\text{comb}_1[k] = -3$$

$$X[3] = -3$$

Alternate Solution:

$$x(t) = \delta(t) * \text{comb}(t) - 4\delta(t-2) * \text{comb}(t) = \text{comb}(t) - 4\,\text{comb}(t-2)$$

$$\text{comb}(t) - 4\,\text{comb}(t-2) \xleftrightarrow{\mathcal{FS}} \text{comb}_1[k] - 4\,\text{comb}_1[k]e^{-j2\pi k(1)(-2)}$$

$$\text{comb}(t) - 4\,\text{comb}(t-2) \xleftrightarrow{\mathcal{FS}} \underbrace{\text{comb}_1[k]}_{=1}[1 - 4e^{j4\pi k}]$$

$$X[k] = 1 - 4e^{j4\pi k} \Rightarrow X[3] = 1 - 4\underbrace{e^{j12\pi}}_{=1} = -3$$

9. (11 pts) Let $x[n] = 5\cos\left(\frac{2\pi(n-3)}{8}\right)$. Let $X[k]$ be its DTFS harmonic function using its fundamental period as the representation time. $X[-25]$ can be expressed as a single complex number in the form, $X[-25] = a + jb$. Find the numerical values of the real numbers, *a* and *b*.

$$\cos\left(\frac{2\pi n}{N_0}\right) \overset{\mathcal{FS}}{\longleftrightarrow} \frac{1}{2}\left(\text{comb}_{N_0}[k-1] + \text{comb}_{N_0}[k+1]\right)$$

$$5\cos\left(\frac{2\pi n}{8}\right) \overset{\mathcal{FS}}{\longleftrightarrow} \frac{5}{2}\left(\text{comb}_8[k-1] + \text{comb}_8[k+1]\right)$$

$$5\cos\left(\frac{2\pi(n-3)}{8}\right) \overset{\mathcal{FS}}{\longleftrightarrow} \frac{5}{2}\left(\text{comb}_8[k-1] + \text{comb}_8[k+1]\right)e^{-j\frac{2\pi k(3)}{8}}$$

$$5\cos\left(\frac{2\pi(n-3)}{8}\right) \overset{\mathcal{FS}}{\longleftrightarrow} \frac{5}{2}\left(\text{comb}_8[k-1] + \text{comb}_8[k+1]\right)e^{-j\frac{3\pi k}{4}}$$

$$X[-25] = \frac{5}{2}\left(\text{comb}_8[-25-1] + \text{comb}_8[-25+1]\right)e^{-j\frac{3\pi(-25)}{4}}$$

$$X[-25] = \frac{5}{2}\left(\underbrace{\text{comb}_8[-26]}_{=0} + \underbrace{\text{comb}_8[-24]}_{=1}\right)e^{j\frac{75\pi}{4}}$$

$$X[-25] = \frac{5}{2}e^{j\frac{75\pi}{4}} = \frac{5}{2}e^{j\left(\frac{75\pi}{4}+2n\pi\right)} = \frac{5}{2}e^{j\left(\frac{75\pi}{4}-18\pi\right)} = \frac{5}{2}e^{j\left(\frac{75\pi}{4}-\frac{72\pi}{4}\right)} = \frac{5}{2}e^{j\frac{3\pi}{4}} = \frac{5}{2}\left(-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right)$$

$$X[-25] = -\frac{5\sqrt{2}}{4} + j\frac{5\sqrt{2}}{4} \Rightarrow a = -\frac{5\sqrt{2}}{4} \text{ and } b = \frac{5\sqrt{2}}{4}$$