# Solution of EE 503 Test #2 S04

1. Complete the following Fourier pairs.

(a) $$5\,\text{rect}\left(\frac{t-2}{4}\right) \xleftrightarrow{\mathcal{F}} 20\,\text{sinc}(4f)e^{-j4\pi f}$$

(b) $$-\frac{9}{2}\left[\delta\left(t+\frac{1}{\pi}\right)+\delta\left(t-\frac{1}{\pi}\right)\right] \xleftrightarrow{\mathcal{F}} -9\cos(2f)$$

$$-9\cos(2f) = -9\frac{e^{j2f}+e^{-j2f}}{2}$$

$$-\frac{9}{2}\left[\delta\left(t+\frac{1}{\pi}\right)+\delta\left(t-\frac{1}{\pi}\right)\right] \xleftrightarrow{\mathcal{F}} -9\frac{e^{j2f}+e^{-j2f}}{2}$$

$$-\frac{9}{2}\left[\delta\left(t+\frac{1}{\pi}\right)+\delta\left(t-\frac{1}{\pi}\right)\right] \xleftrightarrow{\mathcal{F}} -9\cos(2f)$$

(c)

$$2(\text{u}[n]-\text{u}[n-7]) \xleftrightarrow{\mathcal{F}} 2\left(\frac{1}{1-e^{-j2\pi F}}+\frac{1}{2}\text{comb}(F)-\frac{e^{-j14\pi F}}{1-e^{-j2\pi F}}-\underbrace{\frac{e^{-j14\pi F}}{2}\text{comb}(F)}_{=\frac{1}{2}\text{comb}(F)}\right)$$

$$2(\text{u}[n]-\text{u}[n-7]) \xleftrightarrow{\mathcal{F}} 2\frac{1-e^{-j14\pi F}}{1-e^{-j2\pi F}}$$

Alternate Solution:

$$2(\text{u}[n]-\text{u}[n-7]) = 2\,\text{rect}_3[n-3] \xleftrightarrow{\mathcal{F}} 14\,\text{drcl}(F,7)e^{-j6\pi F}$$

(d) $$-\frac{15}{\pi}\sin(4t) \xleftrightarrow{\mathcal{F}} j15[\delta(\omega-4)-\delta(\omega+4)]$$

$$\sin(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} \frac{j}{2}[\delta(f+f_0)-\delta(f-f_0)]$$

$$\sin(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} \frac{j}{2}\left[\delta\left(\frac{\omega}{2\pi}+f_0\right)-\delta\left(\frac{\omega}{2\pi}-f_0\right)\right]$$

$$\sin(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} j\pi[\delta(\omega+2\pi f_0)-\delta(\omega-2\pi f_0)]$$

$$\sin(4t) \xleftrightarrow{\mathcal{F}} j\pi[\delta(\omega+4)-\delta(\omega-4)]$$

$$\frac{15}{\pi}\sin(4t) \xleftrightarrow{\mathcal{F}} j15[\delta(\omega+4)-\delta(\omega-4)]$$

$$-\frac{15}{\pi}\sin(4t) \xleftrightarrow{\mathcal{F}} j15[\delta(\omega-4)-\delta(\omega+4)]$$

2. A CT signal, $\mathrm{x}(t)$, has a CTFT,

$$\mathrm{X}(f)=\begin{cases}|f| \ , \ 0<|f|<1 \\ 0 \ , \ \text{otherwise}\end{cases} .$$

Another CT signal, $\mathrm{x}_p(t)$, is formed by periodically repeating $\mathrm{x}(t)$ with period $T_p=8$. What is the numerical value of the CTFS harmonic function, $\mathrm{X}_p[k]$, at $k=3$, given that $\mathrm{x}_p(t) \xleftrightarrow{\mathcal{FS}} \mathrm{X}_p[k]$ and $\mathrm{X}_p[k]=f_p\,\mathrm{X}(kf_p)$?

$$\mathrm{X}_p[1] = \frac{3}{64} = 0.046875$$

$$\mathrm{X}_p[k]=f_p\,\mathrm{X}(kf_p)=f_p\begin{cases}|kf_p| \ , \ 0<|kf_p|<1 \\ 0 \ , \ \text{otherwise}\end{cases}=\frac{1}{8}\begin{cases}\left|\frac{k}{8}\right| \ , \ 0<\left|\frac{k}{8}\right|<1 \\ 0 \ , \ \text{otherwise}\end{cases}$$

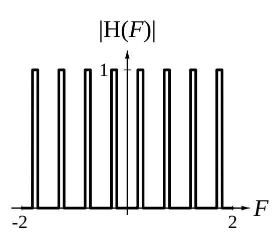
$$\mathrm{X}_p[3]=\frac{1}{8}\begin{cases}\left|\frac{3}{8}\right| \ , \ 0<\left|\frac{3}{8}\right|<1 \\ 0 \ , \ \text{otherwise}\end{cases}=\frac{3}{64}$$

3. Classify the following filters as lowpass, highpass, bandpass, bandstop or none of these.

(a) A filter whose impulse response is $\mathrm{h}(t)=\frac{1}{4}\delta(f)-\mathrm{sinc}(4(t-2))$.

$$\mathrm{H}(f)=\frac{1}{4}-\frac{1}{4}\mathrm{rect}\left(\frac{t}{4}\right)e^{-j4\pi f}$$

Highpass

(b) 


Bandpass

4. In the DSBTC modulator below, the excitation signal is $x_t(t) = \cos(400\pi t)$.

(a) If $m = 1$ and $f_c = 200$ what is the numerical average value of the signal, $y_d(t)$?

$$y_d(t) = [\cos(400\pi t) + 1]\cos^2(400\pi t)$$

$$y_d(t) = [\cos(400\pi t) + 1]\frac{1}{2}[1 + \cos(800\pi t)]$$

$$y_d(t) = \frac{1}{2}[\cos(400\pi t) + 1 + \cos(400\pi t)\cos(800\pi t) + \cos(800\pi t)]$$

Average value of $y_d(t)$ is 0.5.

(b) If $m = 2$ and $f_c = 200$ what is the numerical average value of the signal, $y_d(t)$?

$$y_d(t) = [2\cos(400\pi t) + 1]\cos^2(400\pi t)$$

$$y_d(t) = [2\cos(400\pi t) + 1]\frac{1}{2}[1 + \cos(800\pi t)]$$

$$y_d(t) = \frac{1}{2}[2\cos(400\pi t) + 1 + 2\cos(400\pi t)\cos(800\pi t) + \cos(800\pi t)]$$

Average value of $y_d(t)$ is 0.5.

(c) If $m = 1$ and $f_c = 100$ what is the numerical average value of the signal, $y_d(t)$?

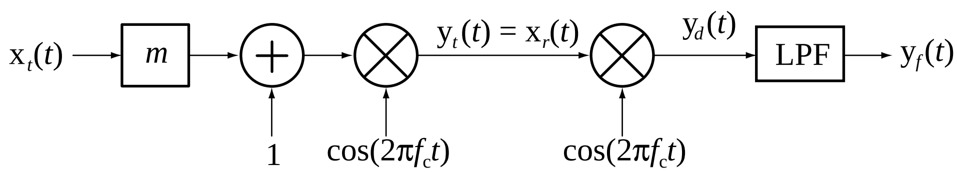
$$y_d(t) = [\cos(400\pi t) + 1]\cos^2(200\pi t)$$

$$y_d(t) = [\cos(400\pi t) + 1]\frac{1}{2}[1 + \cos(400\pi t)]$$

$$y_d(t) = \frac{1}{2}[\cos(400\pi t) + 1 + \cos(400\pi t)\cos(400\pi t) + \cos(400\pi t)]$$

$$y_d(t) = \frac{1}{2}\left[2\cos(400\pi t) + 1 + \frac{1}{2}(1 + \cos(800\pi t))\right]$$

Average value of $y_d(t)$ is 0.75.



5. Find the numerical value of the Nyquist rate for the signal,

$$x(t) = \sin(200\pi t) + \cos(50\pi t)\cos(180\pi t).$$

$$X(f) = \frac{j}{2}\left[\delta(f+100) - \delta(f-100)\right] + \frac{1}{2}\left[\delta(f-25) + \delta(f+25)\right] * \frac{1}{2}\left[\delta(f-90) + \delta(f+90)\right]$$

$$X(f) = \frac{j}{2}\left[\delta(f+100) - \delta(f-100)\right] + \frac{1}{2}\left[\delta(f-115) + \delta(f+65) + \delta(f-65) + \delta(f+115)\right]$$

The highest frequency in the signal is 115 Hz. Therefore the Nyquist rate is 230 Hz.

6. A bandlimited periodic signal is sampled an integer number of times in one fundamental period above its Nyquist rate. The samples are $\{x[0], x[1], x[2], x[3], x[4], x[5], x[6], x[7]\}$. These eight samples are given to the fft (DFT) algorithm in a digital computer and the computer returns this set of eight numbers,

$$\{2, 1-j, 0, 1+j, 0, a, b, c\} \ .$$

(a) Using the complex-conjugate symmetry of all Fourier methods, what are the numerical values of *a*, *b* and *c* ?

$$a = 1-j \ , \ b = 0 \ , \ c = 1+j .$$

(b) What is the numerical value of the first time-domain sample, $x[0]$ ?

$$x[n] = \frac{1}{N_F}\sum_{0}^{N_F-1} X[k] e^{-j2\pi\frac{nk}{N_F}}$$

$$x[0] = \frac{1}{8}\sum_{0}^{7} X[k] = \frac{2+1-j+0+1+j+0+1-j+0+1+j}{8} = \frac{6}{8} = 0.75$$

7. Given a signal, $x[n] = 2\cos\left(\frac{2\pi n}{8}\right) - 4\cos\left(\frac{18\pi n}{8}\right)$,

(a) Find its autocorrelation function, $R_x[m]$.

$$R_x[m] \xleftrightarrow{\mathcal{FS}} X^*[k]X[k]$$

$$X[k] = \left(\text{comb}_8[k-1] + \text{comb}_8[k+1]\right) - 2\left(\text{comb}_8[k-9] + \text{comb}_8[k+9]\right)$$

$$\mathrm{R}_x[m] \xleftrightarrow{\mathcal{FS}} \left[ \left(\mathrm{comb}_8[k-1] + \mathrm{comb}_8[k+1]\right) - 2\left( \underbrace{\mathrm{comb}_8[k-9]}_{=\mathrm{comb}_8[k-1]} + \underbrace{\mathrm{comb}_8[k+9]}_{\mathrm{comb}_8[k+1]} \right) \right] \times$$

$$\left(\mathrm{comb}_8[k-1] + \mathrm{comb}_8[k+1]\right) - 2\left( \underbrace{\mathrm{comb}_8[k-9]}_{=\mathrm{comb}_8[k-1]} + \underbrace{\mathrm{comb}_8[k+9]}_{\mathrm{comb}_8[k+1]} \right)$$

$$\mathrm{R}_x[m] \xleftrightarrow{\mathcal{FS}} \left[ \left(\mathrm{comb}_8[k-1] + \mathrm{comb}_8[k+1]\right) - 2\left(\mathrm{comb}_8[k-1] + \mathrm{comb}_8[k+1]\right) \right] \times$$
$$\left(\mathrm{comb}_8[k-1] + \mathrm{comb}_8[k+1]\right) - 2\left(\mathrm{comb}_8[k-1] + \mathrm{comb}_8[k+1]\right)$$

$$\mathrm{R}_x[m] \xleftrightarrow{\mathcal{FS}} \left[ \left(\mathrm{comb}_8[k-1] + \mathrm{comb}_8[k+1]\right) \right] \times \left(\mathrm{comb}_8[k-1] + \mathrm{comb}_8[k+1]\right)$$

$$\mathrm{comb}_8[k+1]\mathrm{comb}_8[k+1] = \mathrm{comb}_8[k+1] \ \ , \ \ \mathrm{comb}_8[k+1]\mathrm{comb}_8[k-1] = 0$$

$$\mathrm{R}_x[m] \xleftrightarrow{\mathcal{FS}} \left(\mathrm{comb}_8[k-1] + \mathrm{comb}_8[k+1]\right)$$

$$\mathrm{R}_x[m] = 2\cos\left(\frac{2\pi m}{8}\right)$$

(b) What is the average signal power of this signal?

$$P_x = \mathrm{R}_x[0] = 2$$

8. A CT signal with a PSD of $\mathrm{G}_x(f) = 30$ is passed through an ideal lowpass filter with a bandwidth of 100 Hz and a transfer function magnitude of 20 in its passband. What is the signal power of the response signal from this filter?

$$P_y = \int_{-\infty}^{\infty} \mathrm{G}_y(f)\,df$$

$$\mathrm{G}_y(f) = \mathrm{G}_x(f)\left|\mathrm{H}(f)\right|^2 = 30 \times \begin{cases} 20^2 & , \ |f| < 100 \\ 0 & , \ \text{otherwise} \end{cases}$$

$$P_y = 12000 \int_{-100}^{100} df = 2.4 \times 10^6$$