

Student Identification Number _____

By entering my student identification number I affirm that I have neither given help to nor received help from anyone on this test.

ECE 503 Final Examination S04

(Closed Book and Notes, Calculator, Formula Sheet, 8.5 by 11 Sheet of Paper, 2 hours)

If any question or problem is incomplete, ambiguous or contradictory, explain why. If you are correct you will receive full credit on that question or problem.

The phrase “numerical value” appears multiples times on the test. It means that your final answer should be a number, not the mathematical expression which can be reduced to that number. For example, $e^{-j\pi}$ is not a numerical value but -1 is.

1. (6 pts) The Laplace transform of $\frac{d}{dt}[3e^{-5t}u(t)]$ can be expressed in the form, $A + \frac{B}{s+C}$. Find the numerical values of A , B and C .

$A =$ _____ $B =$ _____ $C =$ _____

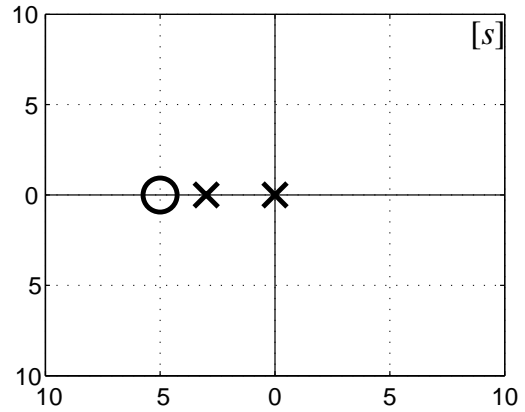
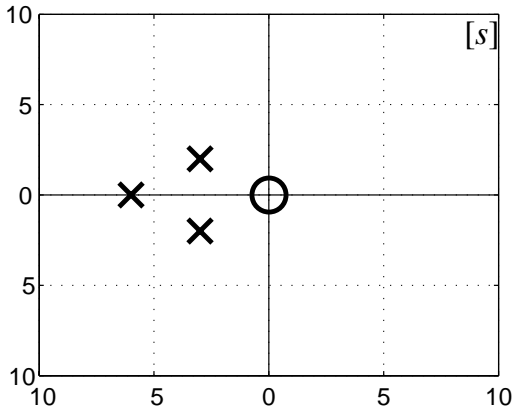
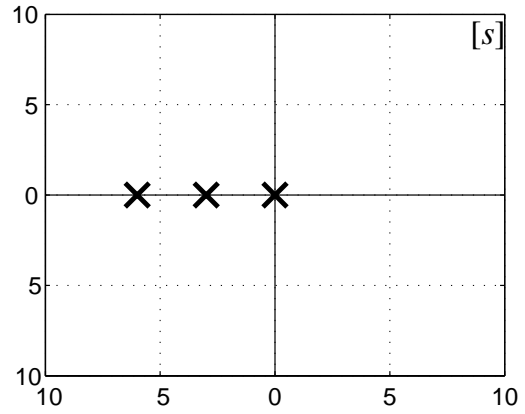
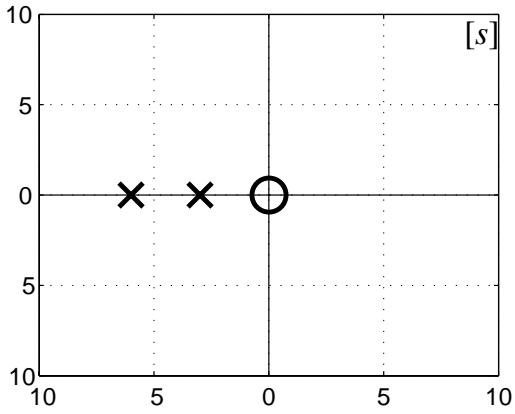
2. (12 pts) The inverse Laplace transform of $\frac{3s+7}{s^2+4s+8}$ can be expressed in the form,

$$e^{at} [A \cos(bt) + C \sin(dt)]$$

where A , C , a , b and d are all real numbers. Find their numerical values.

$A =$ _____ $a =$ _____ $b =$ _____ $C =$ _____ $d =$ _____

3. (4 pts each) Below are some pole-zero plots for the loop transfer function, $T(s)$, of a CT LTI feedback system. Sketch the general shape of the root locus. It should be clear from the sketch which parts are on the real axis (if any), which branches terminate on finite zeros (if any) and the angles of the asymptotes that branches approach as they tend to infinity (if any).



4. (a) (3 pts)

CT filter #1 has transfer function poles at $e^{-j\frac{3\pi}{4}}$ and $e^{+j\frac{3\pi}{4}}$ and a zero at zero.

CT filter #2 has transfer function poles at $e^{-j\frac{7\pi}{12}}$ and $e^{+j\frac{7\pi}{12}}$ and a zero at zero.

They are both bandpass filters. Which one has the smaller half-power or -3 dB bandwidth? Explain your answer.

(b) (3 pts) If you wanted to change either filter in (a) to be a stable highpass filter by adding exactly one pole or one zero, but not both, which one would you add and what is the total range of locations where it could be put? Explain your answer.

5. A unity-gain CT feedback system of the conventional type has a forward path transfer

$$\text{function, } H_1(s) = \frac{s+2}{(s+8)(s+3)}.$$

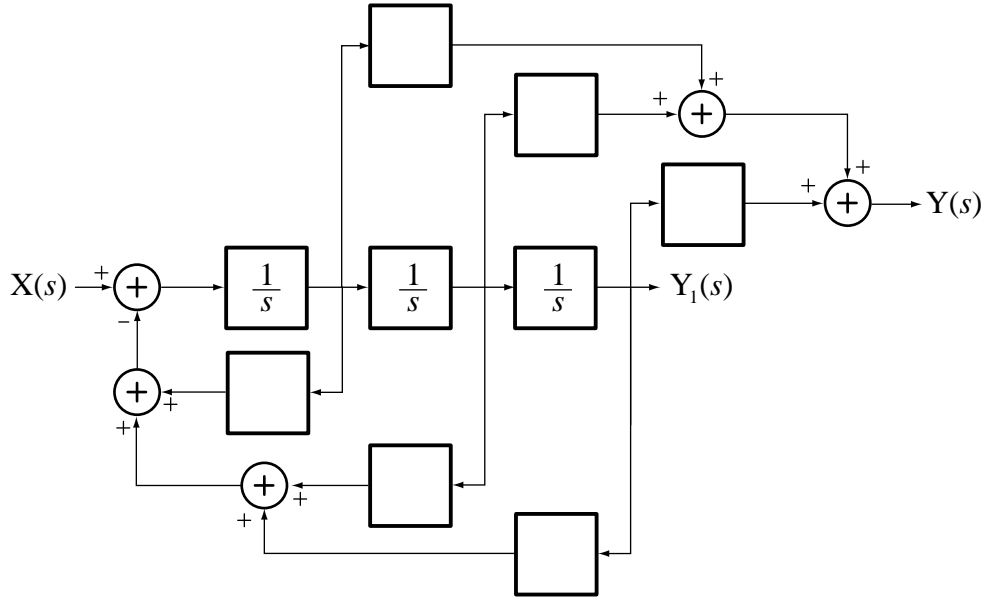
(a) (2 pts) It is excited by a unit step. Will the steady-state error be zero, non-zero and finite, or infinite?

Zero Non-Zero Finite Infinite

(b) (2 pts) It is excited by a unit ramp. Will the steady-state error be zero, non-zero and finite, or infinite?

Zero Non-Zero Finite Infinite

6. (10 pts) A CT LTI system is described by the transfer function, $H(s) = \frac{s(s-4)}{(s+3)(s^2+2s+10)}$. It can be realized in the canonical form block diagram illustrated below. Fill in the values of the gains in the empty blocks. (Put in a zero if a branch is not needed.)



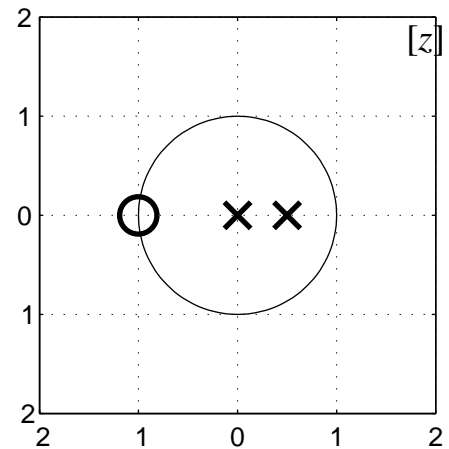
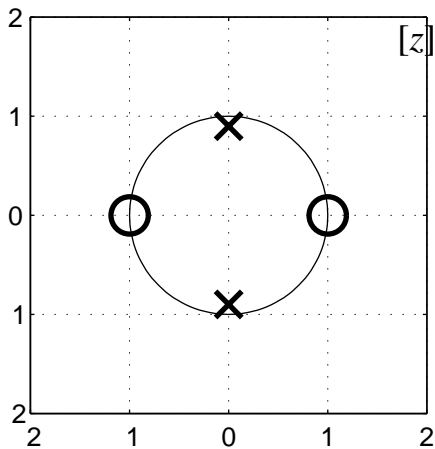
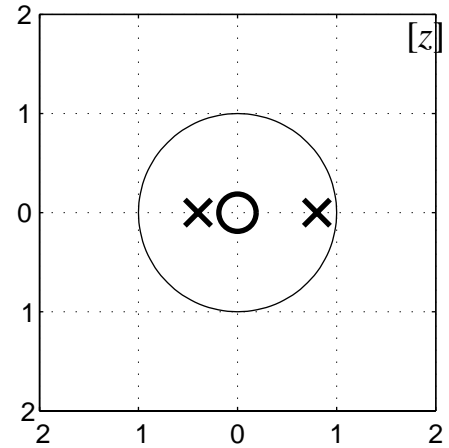
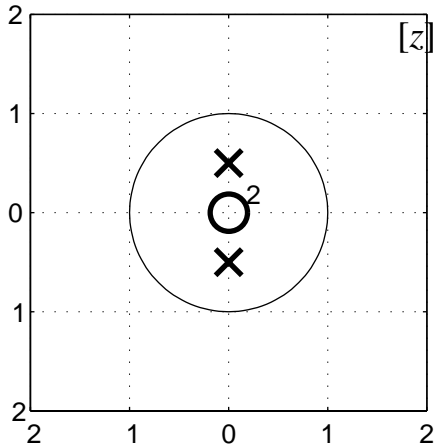
7. (17 pts) The z transform of $x[n] = 4\left(\frac{7}{8}\right)^n u[n] * 6\left(\frac{3}{4}\right)^{n-2} u[n-2]$ can be expressed in the general form, $X(z) = \frac{Az^2 + Bz + C}{z^2 - bz + c}$ where A, B, C, b and c are all real numbers. Find their numerical values. (The “*” is the convolution operator.)

$A =$ _____ $B =$ _____ $C =$ _____ $b =$ _____ $c =$ _____

8. (16 pts) The inverse z transform of $H(z) = \frac{1}{z^2 - 0.3z + 0.02}$ can be written in the form, $h[n] = Aa^{n-b} u[n-b] + Cc^{n-d} u[n-d]$ where A, a, b, C, c and d are all real numbers. Find their numerical values.

$A =$ _____ $a =$ _____ $b =$ _____ $C =$ _____ $c =$ _____ $d =$ _____

9. (4 pts each) Below are some pole-zero plots for the loop transfer function, $T(z)$, of an LTI feedback system. Sketch the general shape of the root locus. It should be clear from the sketch which parts are on the real axis (if any), which branches terminate on finite zeros (if any) and the angles of the asymptotes that branches approach as they tend to infinity (if any). (The superscript, "2", next to a zero means a repeated zero, two zeros at the same place.)



10. (5 pts) A CT filter with a transfer function of the form, $H(s) = \frac{A}{s+a}$, is approximated twice, once by a matched- z digital filter and once by a direct-substitution digital filter. Which one of these digital filters will have a step response that is guaranteed to be zero at time, $n = 0$?

11. (1 pt) A filter designed by the finite difference technique is unstable. It can be stabilized by changing the sampling rate. Should the sampling rate be increased or decreased?

Increased

Decreased

12. (2 pts) Two digital filters are designed to approximate a CT bandpass filter using the impulse invariant technique and the step invariant technique. The two filters are both excited by a unit sequence. Which of them is guaranteed to have a steady-state response of zero?

Impulse Invariant

Step Invariant

13. (2 pts) What generally happens to all poles and zeros of a digital filter as the sampling rate is increased? That is, what point in the z plane do they all approach?

14. (1 pt) Which of the digital filter design techniques we studied maps the s to z plane one-to-one and also the z to s plane one-to-one? That is, the transformation is single-valued in both directions.

Solution of EE 503 Final Examination S04 #1

1. (6 pts) The Laplace transform of $\frac{d}{dt}[3e^{-5t}u(t)]$ can be expressed in the form, $A + \frac{B}{s+C}$. Find the numerical values of A , B and C .

$$A = 3 \quad B = -15 \quad C = 5$$

$$3e^{-5t}u(t) \xrightarrow{\mathcal{L}} \frac{3}{s+5}$$

$$\frac{d}{dt}[3e^{-5t}u(t)] \xrightarrow{\mathcal{L}} \frac{3s}{s+5} = 3 - \frac{15}{s+5}$$

2. (12 pts) The inverse Laplace transform of $\frac{3s+7}{s^2+4s+8}$ can be expressed in the form,

$$e^{at} [A \cos(bt) + C \sin(dt)]$$

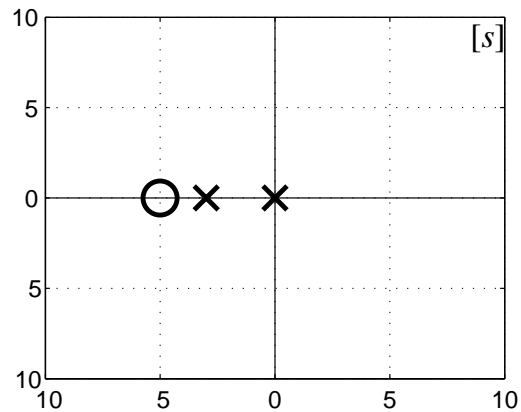
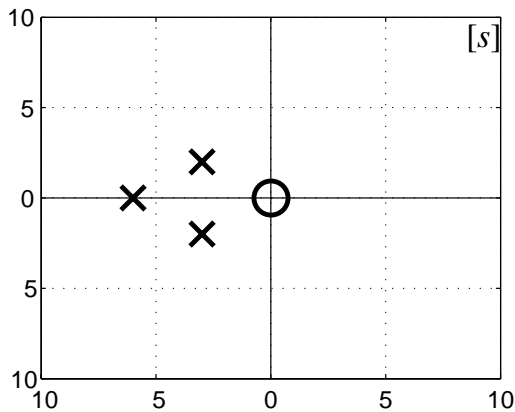
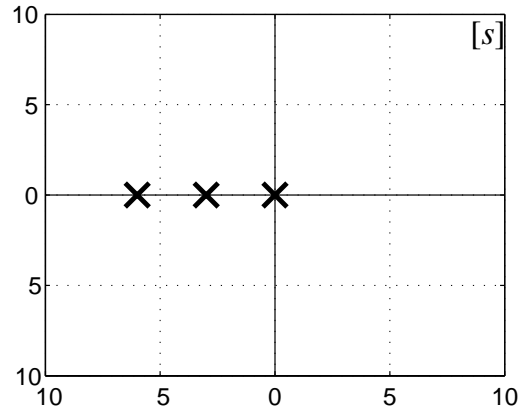
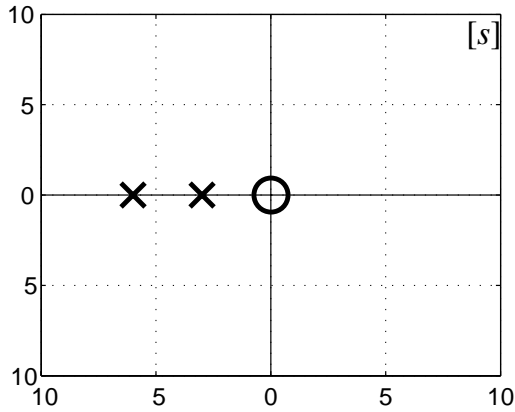
where A , C , a , c and d are all real numbers. Find their numerical values.

$$A = 3 \quad a = -2 \quad b = 2 \quad C = 1/2 \quad d = 2$$

$$\frac{3s+7}{s^2+4s+8} = 3 \frac{s+\frac{7}{3}}{(s+2)^2+4} = 3 \left[\frac{s+2}{(s+2)^2+4} + \frac{1}{6} \frac{2}{(s+2)^2+4} \right]$$

$$e^{-2t} \left[3 \cos(2t) + \frac{1}{2} \sin(2t) \right] \xrightarrow{\mathcal{L}} 3 \left[\frac{s+2}{(s+2)^2+4} + \frac{1}{6} \frac{2}{(s+2)^2+4} \right]$$

3. (4 pts each) Below are some pole-zero plots for the loop transfer function, $T(s)$, of a CT LTI feedback system. Sketch the general shape of the root locus. It should be clear from the sketch which parts are on the real axis (if any), which branches terminate on finite zeros (if any) and the angles of the asymptotes that branches approach as they tend to infinity (if any).



4. (a) (3 pts)

CT filter #1 has transfer function poles at $e^{-j\frac{3\pi}{4}}$ and $e^{+j\frac{3\pi}{4}}$ and a zero at zero.

CT filter #2 has transfer function poles at $e^{-j\frac{7\pi}{12}}$ and $e^{+j\frac{7\pi}{12}}$ and a zero at zero.

They are both bandpass filters. Which one has the smaller half-power or -3 dB bandwidth? Explain your answer.

Filter #2 has the smaller bandwidth because the poles are closer to the ω axis indicating a higher Q, lower damping factor and smaller bandwidth.

(b) (3 pts) If you wanted to change either filter in (a) to be a stable highpass filter by adding exactly one pole or one zero, but not both, which one would you add and what is the total range of locations where it could be put? Explain your answer.

Add a zero anywhere on the real axis.

5. A unity-gain CT feedback system of the conventional type has a forward path transfer function, $H_1(s) = \frac{s+2}{(s+8)(s+3)}$.

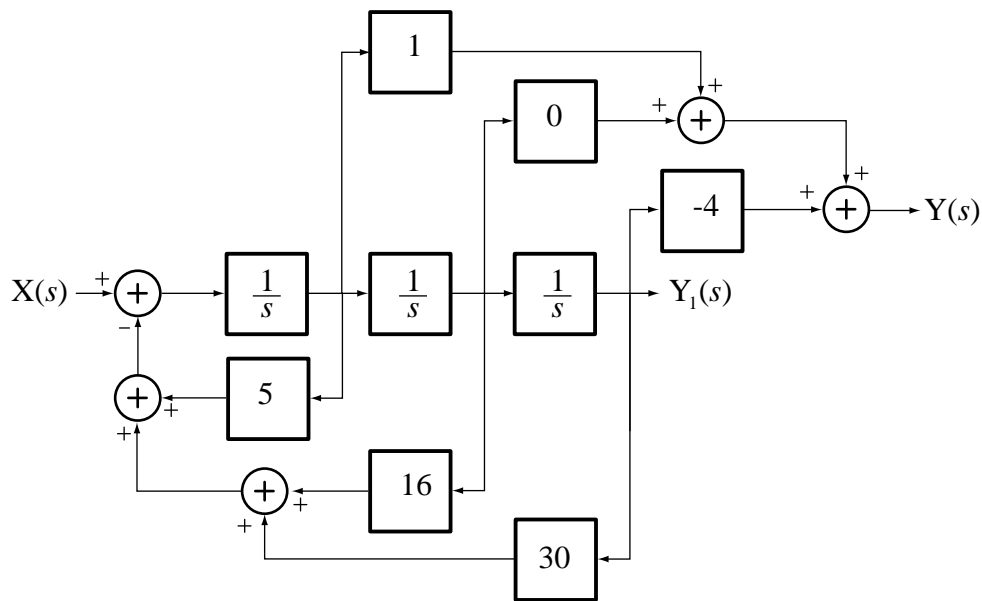
(a) (2 pts) It is excited by a unit step. Will the steady-state error be zero, non-zero and finite, or infinite?

Non-Zero Finite

(b) (2 pts) It is excited by a unit ramp. Will the steady-state error be zero, non-zero and finite, or infinite?

Infinite

6. (10 pts) A CT LTI system is described by the transfer function, $H(s) = \frac{s(s-4)}{(s+3)(s^2+2s+10)}$. It can be realized in the canonical form block diagram illustrated below. Fill in the values of the gains in the empty blocks. (Put in a zero if a branch is not needed.)



$$H(s) = \frac{s(s-4)}{(s+3)(s^2+2s+10)} = \frac{s^2 - 4s}{s^3 + 5s^2 + 16s + 30}$$

7. (17 pts) The z transform of $x[n] = 4\left(\frac{7}{8}\right)^n u[n] * 6\left(\frac{3}{4}\right)^{n-2} u[n-2]$ can be expressed in the general form, $X(z) = \frac{Az^2 + Bz + C}{z^2 - bz + c}$ where A, B, C, b and c are all real numbers. Find their numerical values. (The “*” is the convolution operator.)

$$A = 0 \quad B = 0 \quad C = 24 \quad b = -1.625 \quad c = 0.65625$$

$$4\left(\frac{7}{8}\right)^n u[n] * 6\left(\frac{3}{4}\right)^{n-2} u[n-2] \xrightarrow{z} 4 \frac{z}{z - \frac{7}{8}} \times 6z^{-2} \frac{z}{z - \frac{3}{4}}$$

$$4\left(\frac{7}{8}\right)^n u[n] * 6\left(\frac{3}{4}\right)^{n-2} u[n-2] \xrightarrow{z} 24 \frac{1}{\left(z - \frac{7}{8}\right)\left(z - \frac{3}{4}\right)}$$

$$4\left(\frac{7}{8}\right)^n u[n] * 6\left(\frac{3}{4}\right)^{n-2} u[n-2] \xrightarrow{z} \frac{24}{z^2 - \frac{13}{8}z + \frac{21}{32}} = \frac{24}{z^2 - 1.625z + 0.65625}$$

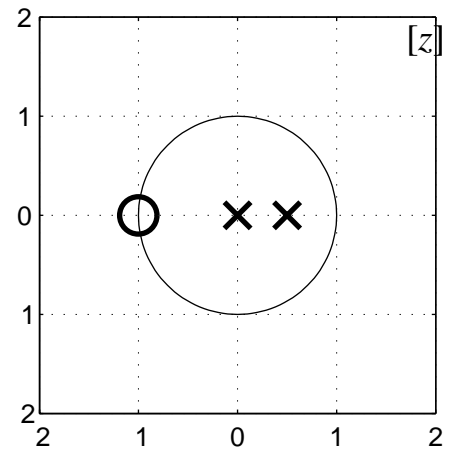
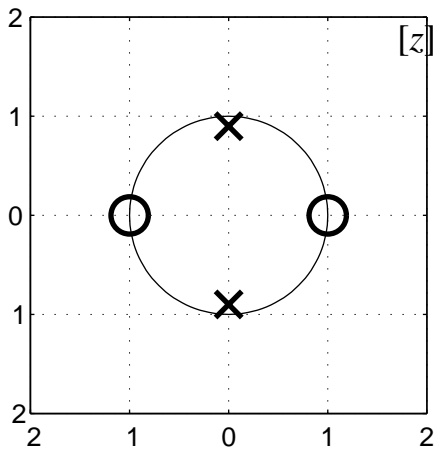
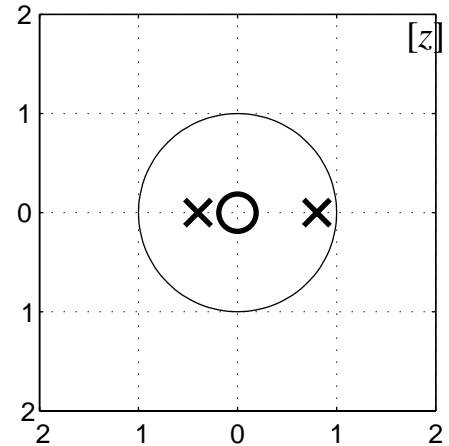
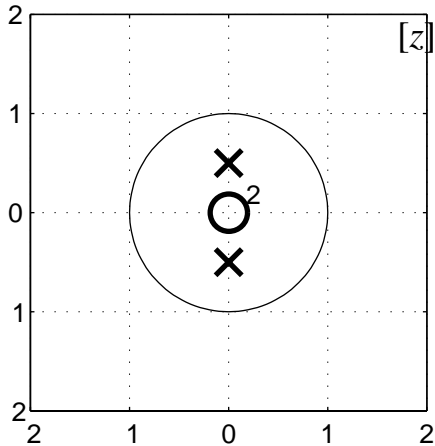
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$$A = 10 \quad a = 0.2 \quad b = 1 \quad C = -10 \quad c = 0.1 \quad d = 1$$

$$\frac{1}{z^2 - 0.3z + 0.02} = \frac{1}{(z - 0.2)(z - 0.1)} = \frac{10}{z - 0.2} - \frac{10}{z - 0.1}$$

$$10(0.2)^{n-1} u[n-1] - 10(0.1)^{n-1} u[n-1] \xrightarrow{z} \frac{10}{z - 0.2} - \frac{10}{z - 0.1}$$

9. (4 pts each) Below are some pole-zero plots for the loop transfer function, $T(z)$, of an LTI feedback system. Sketch the general shape of the root locus. It should be clear from the sketch which parts are on the real axis (if any), which branches terminate on finite zeros (if any) and the angles of the asymptotes that branches approach as they tend to infinity (if any). (The superscript, "2", next to a zero means a repeated zero, two zeros at the same place.)



10. (5 pts) A CT filter with a transfer function of the form, $H(s) = \frac{A}{s+a}$, is approximated twice, once by a matched- z digital filter and once by a direct-substitution digital filter. Which one of these digital filters will have a step response that is guaranteed to be zero at time, $n = 0$?

Direct Substitution

Matched- z $H(z) = \frac{A}{1 - z^{-1}e^{-aT_s}} = \frac{Az}{z - e^{-aT_s}} \Rightarrow h[n] = Ae^{-anT_s} u[n]$

Direct Substitution $H(z) = \frac{A}{z - e^{-aT_s}} \Rightarrow h[n] = Ae^{-a(n-1)T_s} u[n-1]$

11. (1 pt) A filter designed by the finite difference technique is unstable. It can be stabilized by changing the sampling rate. Should the sampling rate be increased or decreased?

Increased

12. (2 pts) Two digital filters are designed to approximate a CT bandpass filter using the impulse invariant technique and the step invariant technique. The two filters are both excited by a unit sequence. Which of them is guaranteed to have a steady-state response of zero?

Step Invariant

13. (2 pts) What generally happens to all poles and zeros of a digital filter as the sampling rate is increased? That is, what point in the z plane do they all approach?

They all approach the $z = 1$ point.

14. (1 pt) Which of the digital filter design techniques we studied maps the s to z plane one-to-one and also the z to s plane one-to-one? That is, the transformation is single-valued in both directions.

Bilinear z Transform

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ECE 503 Final Examination S04

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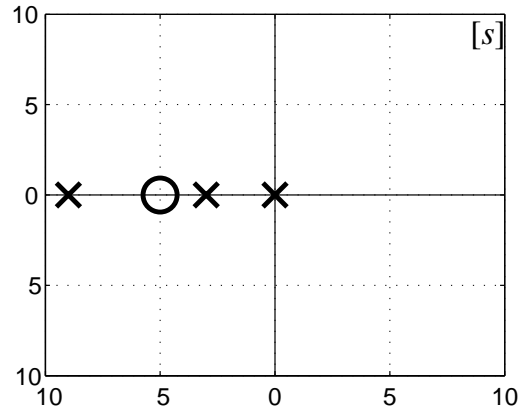
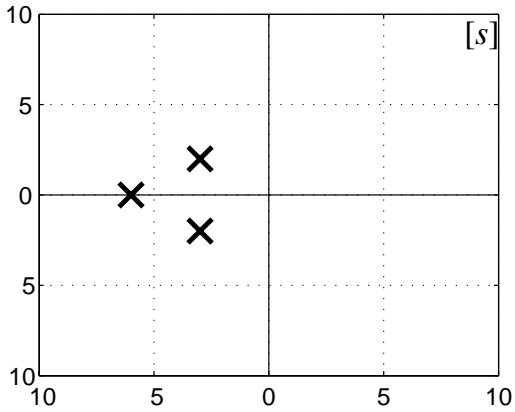
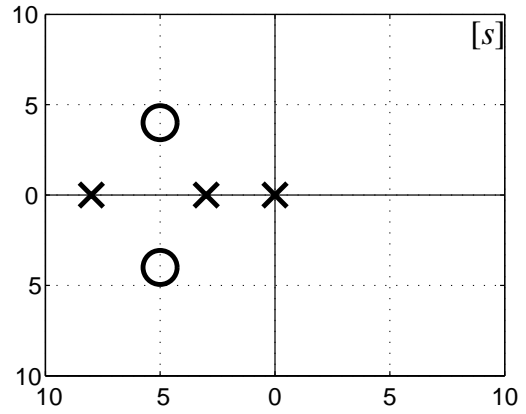
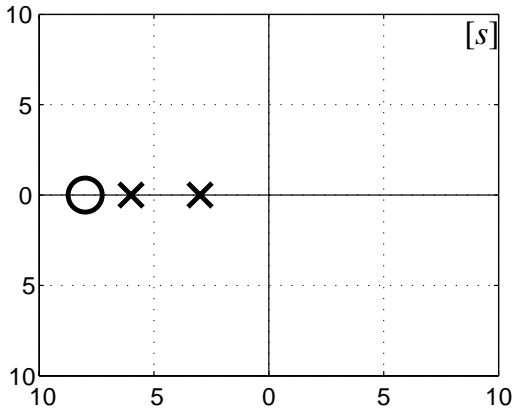
2. (12 pts) The inverse Laplace transform of $\frac{2s+1}{s^2+4s+8}$ can be expressed in the form,

$$e^{at} [A \cos(bt) + C \sin(dt)]$$

where A , C , a , b and d are all real numbers. Find their numerical values.

$A =$ _____ $a =$ _____ $b =$ _____ $C =$ _____ $d =$ _____

3. (4 pts each) Below are some pole-zero plots for the loop transfer function, $T(s)$, of a CT LTI feedback system. Sketch the general shape of the root locus. It should be clear from the sketch which parts are on the real axis (if any), which branches terminate on finite zeros (if any) and the angles of the asymptotes that branches approach as they tend to infinity (if any).



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They are both bandpass filters. Which one has the smaller half-power or -3 dB bandwidth? Explain your answer.

(b) (3 pts) If you wanted to change either filter in (a) to be a stable highpass filter by adding exactly one pole or one zero, but not both, which one would you add and what is the total range of locations where it could be put? Explain your answer.

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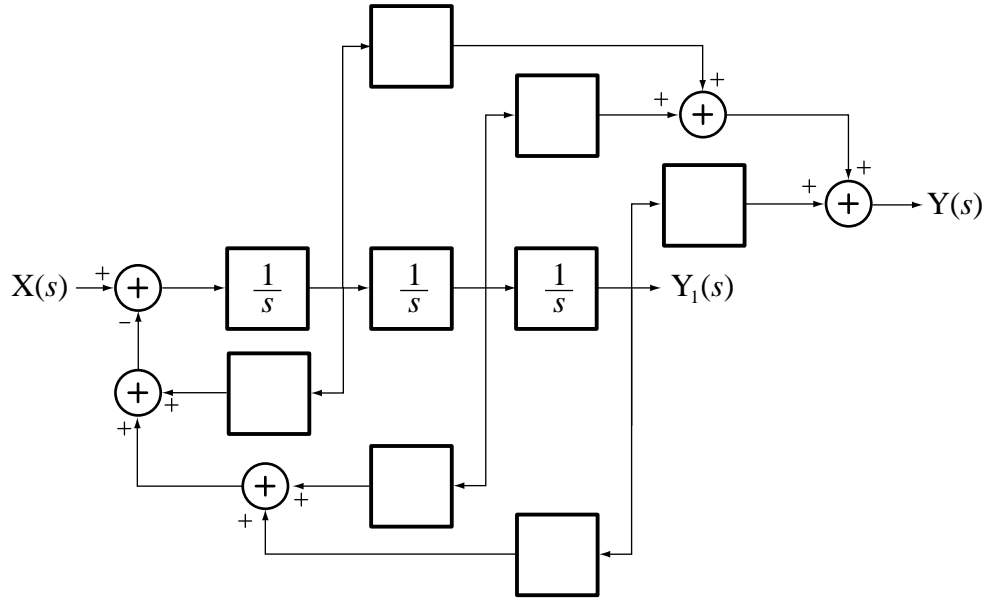
(a) (2 pts) It is excited by a unit step. Will the steady-state error be zero, non-zero and finite, or infinite?

Zero Non-Zero Finite Infinite

(b) (2 pts) It is excited by a unit ramp. Will the steady-state error be zero, non-zero and finite, or infinite?

Zero Non-Zero Finite Infinite

6. (10 pts) A CT LTI system is described by the transfer function, $H(s) = \frac{s^2 + 6}{(s+1)(s^2 + 4s + 12)}$. It can be realized in the canonical form block diagram illustrated below. Fill in the values of the gains in the empty blocks. (Put in a zero if a branch is not needed.)



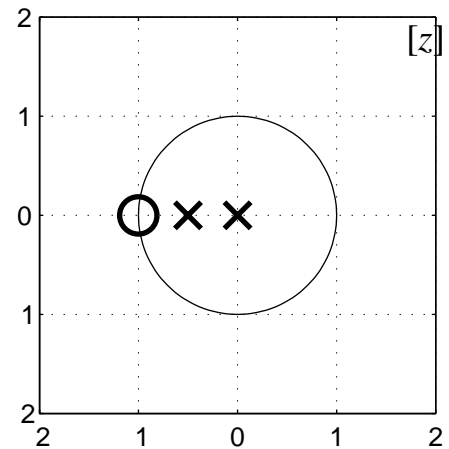
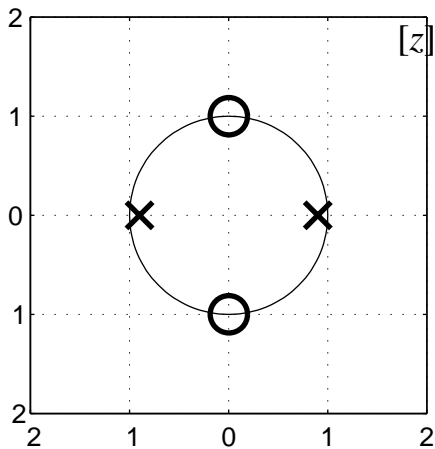
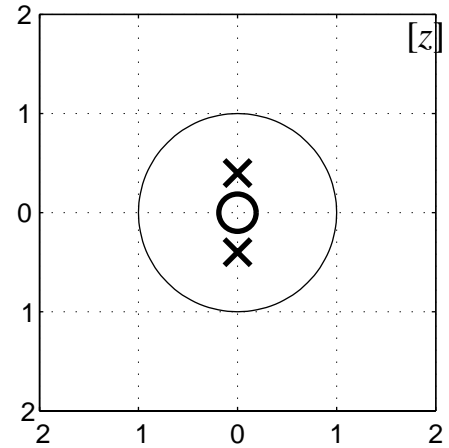
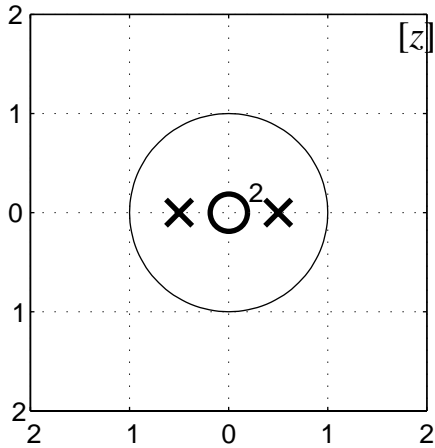
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$A = \underline{\hspace{2cm}}$ $B = \underline{\hspace{2cm}}$ $C = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$ $c = \underline{\hspace{2cm}}$

8. (16 pts) The inverse z transform of $H(z) = \frac{1}{z^2 - 0.5z + 0.04}$ can be written in the form, $h[n] = Aa^{n-b} u[n-b] + Cc^{n-d} u[n-d]$ where A, a, b, C, c and d are all real numbers. Find their numerical values.

$A =$ _____ $a =$ _____ $b =$ _____ $C =$ _____ $c =$ _____ $d =$ _____

9. (4 pts each) Below are some pole-zero plots for the loop transfer function, $T(z)$, of an LTI feedback system. Sketch the general shape of the root locus. It should be clear from the sketch which parts are on the real axis (if any), which branches terminate on finite zeros (if any) and the angles of the asymptotes that branches approach as they tend to infinity (if any). (The superscript, "2", next to a zero means a repeated zero, two zeros at the same place.)



10. (5 pts) A CT filter with a transfer function of the form, $H(s) = \frac{A}{s+a}$, is approximated twice, once by a matched- z digital filter and once by a direct-substitution digital filter. Which one of these digital filters will have a step response that is guaranteed to be zero at time, $n = 0$?

11. (1 pt) A filter designed by the finite difference technique is stable. It can be made unstable by changing the sampling rate. Should the sampling rate be increased or decreased?

Increased

Decreased

12. (2 pts) Two digital filters are designed to approximate a CT bandpass filter using the impulse invariant technique and the step invariant technique. The two filters are both excited by a unit sequence. Which of them is not guaranteed to have a steady-state response of zero?

Impulse Invariant

Step Invariant

13. (2 pts) What generally happens to all poles and zeros of a digital filter as the sampling rate is increased? That is, what point in the z plane do they all approach?

14. (1 pt) Which of the digital filter design techniques we studied maps the s to z plane one-to-one and also the z to s plane one-to-one? That is, the transformation is single-valued in both directions.

Solution of EE 503 Final Examination S04 #2

1. (6 pts) The Laplace transform of $\frac{d}{dt}[5e^{-3t}u(t)]$ can be expressed in the form, $A + \frac{B}{s+C}$. Find the numerical values of A , B and C .

$$A = 5 \quad B = -15 \quad C = 3$$

$$5e^{-3t}u(t) \xrightarrow{\mathcal{L}} \frac{5}{s+3}$$

$$\frac{d}{dt}[5e^{-3t}u(t)] \xrightarrow{\mathcal{L}} \frac{5s}{s+3} = 5 - \frac{15}{s+3}$$

2. (12 pts) The inverse Laplace transform of $\frac{2s+1}{s^2+4s+8}$ can be expressed in the form,

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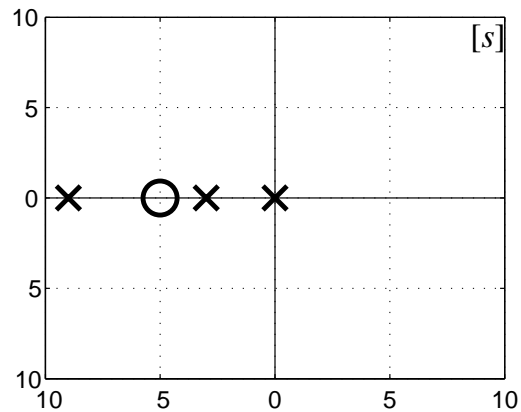
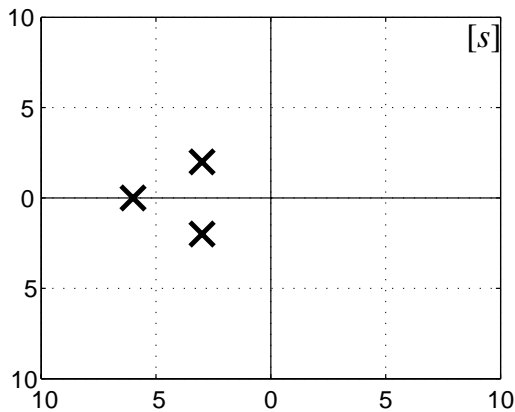
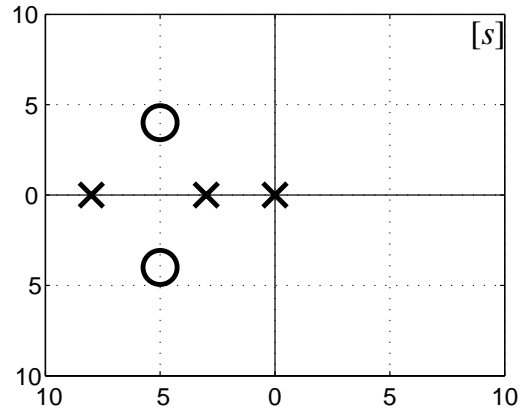
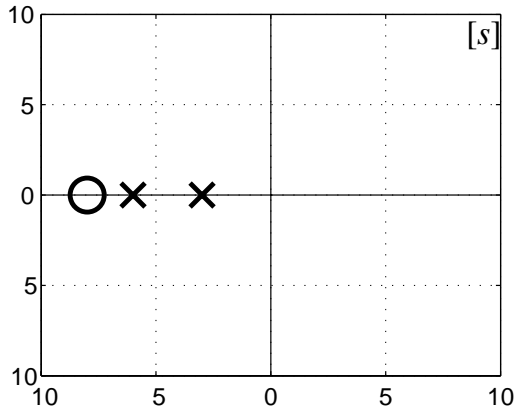
where A , C , a , c and d are all real numbers. Find their numerical values.

$$A = 2 \quad a = -2 \quad b = 2 \quad C = -3/2 \quad d = 2$$

$$\frac{2s+1}{s^2+4s+8} = 2 \frac{s+\frac{1}{2}}{(s+2)^2+4} = 2 \left[\frac{s+2}{(s+2)^2+4} - \frac{3}{4} \frac{2}{(s+2)^2+4} \right]$$

$$e^{-2t} \left[2 \cos(2t) - \frac{3}{2} \sin(2t) \right] \xrightarrow{\mathcal{L}} 2 \left[\frac{s+2}{(s+2)^2+4} - \frac{3}{4} \frac{2}{(s+2)^2+4} \right]$$

3. (4 pts each) Below are some pole-zero plots for the loop transfer function, $T(s)$, of a CT LTI feedback system. Sketch the general shape of the root locus. It should be clear from the sketch which parts are on the real axis (if any), which branches terminate on finite zeros (if any) and the angles of the asymptotes that branches approach as they tend to infinity (if any).



4. (a) (3 pts)

CT filter #1 has transfer function poles at $e^{-j\frac{3\pi}{4}}$ and $e^{+j\frac{3\pi}{4}}$ and a zero at zero.

CT filter #2 has transfer function poles at $e^{-j\frac{7\pi}{12}}$ and $e^{+j\frac{7\pi}{12}}$ and a zero at zero.

They are both bandpass filters. Which one has the larger half-power or -3 dB bandwidth? Explain your answer.

Filter #1 has the larger bandwidth because the poles are farther from the ω axis indicating a lower Q, higher damping factor and larger bandwidth.

(b) (3 pts) If you wanted to change either filter in (a) to be a stable highpass filter by adding exactly one pole or one zero, but not both, which one would you add and what is the total range of locations where it could be put? Explain your answer.

Add a zero anywhere on the real axis.

5. A unity-gain CT feedback system of the conventional type has a forward path transfer function, $H_1(s) = \frac{s+2}{s(s+8)(s+3)}$.

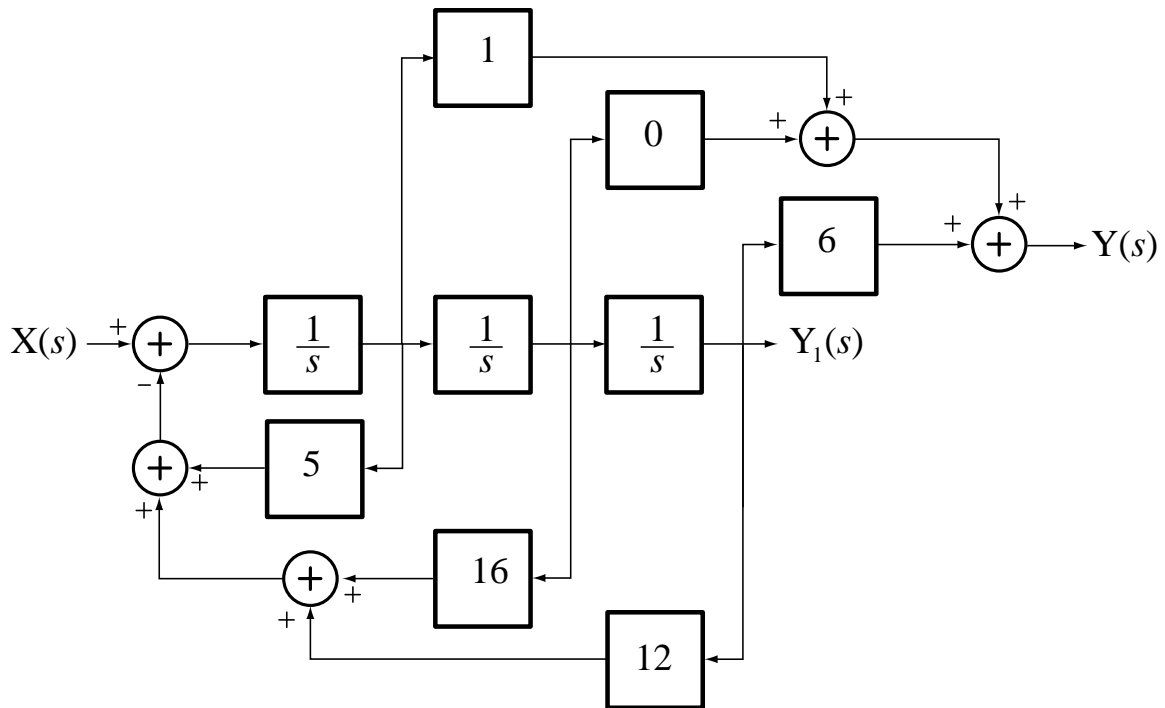
(a) (2 pts) It is excited by a unit step. Will the steady-state error be zero, non-zero and finite, or infinite?

Zero

(b) (2 pts) It is excited by a unit ramp. Will the steady-state error be zero, non-zero and finite, or infinite?

Non-zero Finite

6. (10 pts) A CT LTI system is described by the transfer function, $H(s) = \frac{s^2 + 6}{(s+1)(s^2 + 4s + 12)}$. It can be realized in the canonical form block diagram illustrated below. Fill in the values of the gains in the empty blocks. (Put in a zero if a branch is not needed.)



$$H(s) = \frac{s^2 + 6}{(s+1)(s^2 + 4s + 12)} = \frac{s^2 + 6}{s^3 + 5s^2 + 16s + 12}$$

7. (17 pts) The z transform of $x[n] = 5\left(\frac{7}{10}\right)^n u[n] * 7\left(\frac{3}{5}\right)^{n-1} u[n-1]$ can be expressed in the general form, $X(z) = \frac{Az^2 + Bz + C}{z^2 - bz + c}$ where A, B, C, b and c are all real numbers. Find their numerical values. (The “*” is the convolution operator.)

$$A = 0 \quad B = 35 \quad C = 0 \quad b = -1.3 \quad c = 0.42$$

$$5\left(\frac{7}{10}\right)^n u[n] * 7\left(\frac{3}{5}\right)^{n-1} u[n-1] \xrightarrow{z} 5 \frac{z}{z - \frac{7}{10}} \times 7z^{-1} \frac{z}{z - \frac{3}{5}}$$

$$5\left(\frac{7}{10}\right)^n u[n] * 7\left(\frac{3}{5}\right)^{n-1} u[n-1] \xrightarrow{z} 35 \frac{z}{\left(z - \frac{7}{10}\right)\left(z - \frac{3}{5}\right)}$$

$$5\left(\frac{7}{10}\right)^n u[n] * 7\left(\frac{3}{5}\right)^{n-1} u[n-1] \xrightarrow{z} \frac{35z}{z^2 - \frac{13}{10}z + \frac{21}{50}} = \frac{35z}{z^2 - 1.3z + 0.42}$$

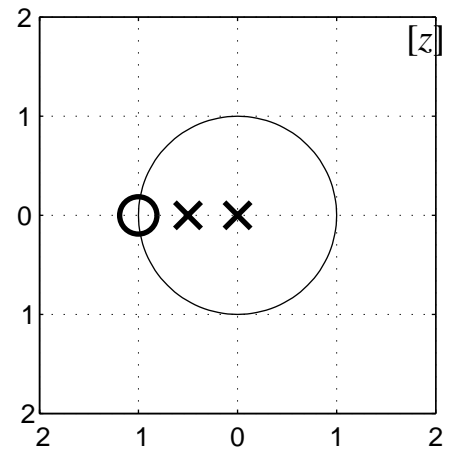
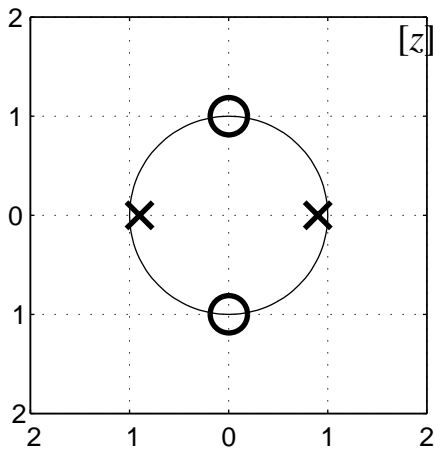
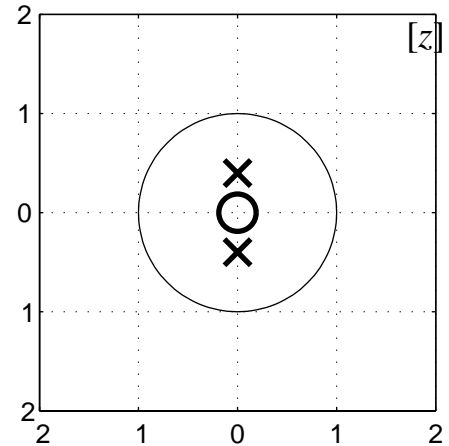
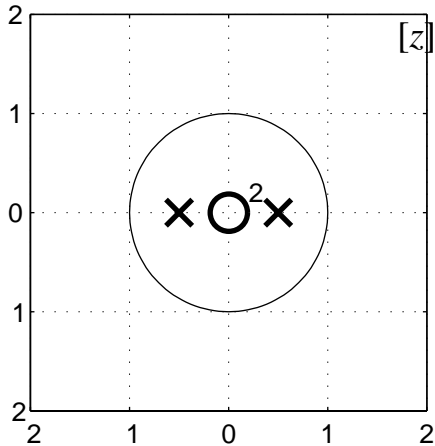
8. (16 pts) The inverse z transform of $H(z) = \frac{1}{z^2 - 0.5z + 0.04}$ can be written in the form, $h[n] = Aa^{n-b} u[n-b] + Cc^{n-d} u[n-d]$ where A, a, b, C, c and d are all real numbers. Find their numerical values.

$$A = \frac{10}{3} \quad a = 0.4 \quad b = 1 \quad C = -\frac{10}{3} \quad c = 0.1 \quad d = 1$$

$$\frac{1}{z^2 - 0.5z + 0.04} = \frac{1}{(z - 0.4)(z - 0.1)} = \frac{\frac{10}{3}}{z - 0.4} - \frac{\frac{10}{3}}{z - 0.1}$$

$$\frac{10}{3}(0.4)^{n-1} u[n-1] - \frac{10}{3}(0.1)^{n-1} u[n-1] \xrightarrow{z} \frac{\frac{10}{3}}{z - 0.2} - \frac{\frac{10}{3}}{z - 0.1}$$

9. (4 pts each) Below are some pole-zero plots for the loop transfer function, $T(z)$, of an LTI feedback system. Sketch the general shape of the root locus. It should be clear from the sketch which parts are on the real axis (if any), which branches terminate on finite zeros (if any) and the angles of the asymptotes that branches approach as they tend to infinity (if any). (The superscript, "2", next to a zero means a repeated zero, two zeros at the same place.)



10. (5 pts) A CT filter with a transfer function of the form, $H(s) = \frac{A}{s+a}$, is approximated twice, once by a matched- z digital filter and once by a direct-substitution digital filter. Which one of these digital filters will have a step response that is guaranteed to be zero at time, $n = 0$?

Direct Substitution

Matched- z $H(z) = \frac{A}{1 - z^{-1}e^{-aT_s}} = \frac{Az}{z - e^{-aT_s}} \Rightarrow h[n] = Ae^{-anT_s} u[n]$

Direct Substitution $H(z) = \frac{A}{z - e^{-aT_s}} \Rightarrow h[n] = Ae^{-a(n-1)T_s} u[n-1]$

11. (1 pt) A filter designed by the finite difference technique is stable. It can be made unstable by changing the sampling rate. Should the sampling rate be increased or decreased?

Decreased

12. (2 pts) Two digital filters are designed to approximate a CT bandpass filter using the impulse invariant technique and the step invariant technique. The two filters are both excited by a unit sequence. Which of them is not guaranteed to have a steady-state response of zero?

Impulse Invariant

13. (2 pts) What generally happens to all poles and zeros of a digital filter as the sampling rate is increased? That is, what point in the z plane do they all approach?

They all approach the $z = 1$ point.

14. (1 pt) Which of the digital filter design techniques we studied maps the s to z plane one-to-one and also the z to s plane one-to-one? That is, the transformation is single-valued in both directions.

Bilinear z Transform