Solution of EE 503 Test #2 Su03

1. (4 pts) A CT system is defined by $y'(t) = x(t)$.

   Is it BIBO stable? No

   The impulse response is the unit step, $u(t)$. This function is not absolutely integrable. Therefore the system is not BIBO stable. A constant excitation produces an unbounded response.

2. (4 pts) A CT system is defined by $y''(t) + y(t) = x(t)$.

   Is it BIBO stable? No

   The impulse response is a sinusoid after time, $t = 0$. This function is not absolutely integrable. Therefore the system is not BIBO stable. The response of this system to a sinusoid of 1 radian per second is unbounded.

3. (4 pts) A DT system has an impulse response, $h[n] = \sum_{m=0}^{\infty} \delta[n-2m] - \delta[n-1-2m]$.

   Is it causal? Yes

   Is it BIBO stable? No

   Its impulse response is not absolutely summable. Therefore it is not BIBO stable.

4. (5 pts) Sketch the impulse response of the system described by

   $y[n] + y[n-1] = x[n] - x[n-1]$ where $x[n]$ is the excitation and $y[n]$ is the response.

First find the response of the system, $h_0[n] + h_0[n-1] = \delta[n]$. Then $h[n] = h_0[n] - h_0[n-1]$.

By the usual method of finding the homogeneous solution, the eigenvalue is -1, $h_0[0] = 1$ and

$$h_0[n] = (-1)^n u[n] \Rightarrow h[n] = (-1)^n u[n] - (-1)^{n-1} u[n-1]$$

$$h[n] = \delta[n] + (-1)^n u[n-1] - (-1)^{n-1} u[n-1] = \delta[n] + (-1)^n \left\{1 - (-1)^{-1}\right\} u[n-1]$$

$$h[n] = \delta[n] + 2(-1)^n u[n-1]$$

5. (12 pts) A CT system has an impulse response, $\mathrm{h}(t)=\mathrm{rect}\left(\frac{t-1}{2}\right)$. It is excited by a signal, $\mathrm{x}(t)=\cos(2\pi f_0 t)\mathrm{u}(t)$. For any time, $t>2$, the response of the system is zero. Find two different possible numerical values for f_0.

The response is

$$\mathrm{x}(t)*\mathrm{h}(t)=\int_{-\infty}^{\infty}\cos(2\pi f_0\tau)\mathrm{u}(\tau)\mathrm{rect}\left(\frac{t-\tau-1}{2}\right)d\tau$$

$$\mathrm{x}(t)*\mathrm{h}(t)=\int_{0}^{\infty}\cos(2\pi f_0\tau)\mathrm{rect}\left(\frac{t-\tau-1}{2}\right)d\tau$$

For $t>2$, $\mathrm{rect}\left(\frac{t-\tau-1}{2}\right)$ is zero for $\frac{t-\tau-1}{2}<-\frac{1}{2}$ or for $\tau>t$. It is one for $t-2<\tau<t$. And it is zero for $\tau<t-2$. Therefore the integral can be written as

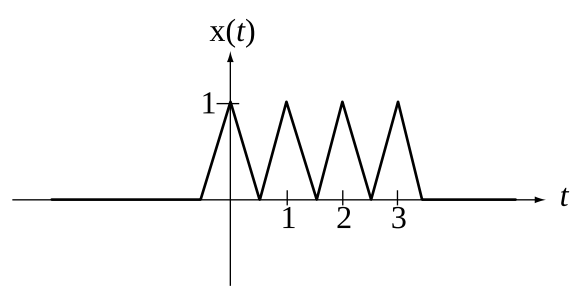
$$\mathrm{x}(t)*\mathrm{h}(t)=\int_{t-2}^{t}\cos(2\pi f_0\tau)d\tau$$

If this integral is to be equal to zero, the time range, $t-2$ to t, which is 2 seconds long must be an integer number of periods of the cosine function. So

$$mT_0=\frac{m}{f_0}=2\Rightarrow f_0=\frac{m}{2}\quad,\quad m \text{ any integer.}$$

This result is also easily seen by sketching the graphical convolution, letting the rectangle function be the function flipped and shifted.

6. (5 pts) Sketch the signal, $\mathrm{x}(t)=\mathrm{tri}(2t)*\sum_{n=0}^{3}\delta(t-n)$ over the time interval, $-5<t<5$.



7. (4 pts) If the response of an LTI system to the excitation, $\mathrm{u}[n]-\mathrm{u}[n-4]$ is

$$\mathrm{y}[n]=\begin{cases}\mathrm{ramp}[n+1] & , \quad n<4 \\ 4 & , \quad n\ge 4\end{cases}$$

what is the response of this same system to the excitation, $\delta[n]-\delta[n-4]$?

$\delta[n]-\delta[n-4]$ is the first backward difference of $\mathrm{u}[n]-\mathrm{u}[n-4]$. Therefore the response is the first backward difference of

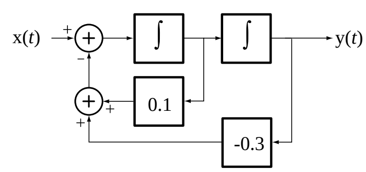
$$\mathrm{y}[n]=\begin{cases}\mathrm{ramp}[n+1] & , \quad n<4 \\ 4 & , \quad n\ge 4\end{cases}$$

which is $\mathrm{u}[n]-\mathrm{u}[n-4]$.

8. (4 pts) A CT system has a response to a unit step of $\mathrm{h}_{-1}(t)=(1-e^{-t})\mathrm{u}(t)$. What is its impulse response, $\mathrm{h}(t)$? The impulse response is the derivative of the step response. Therefore

$$\mathrm{h}(t)=\frac{d}{dt}(1-e^{-t})\mathrm{u}(t)=e^{-t}\,\mathrm{u}(t)$$

9. (10 pts) Write the differential equation for the system described by the block diagram below. (Be sure to observe the signs on the summers.)



Is this system stable? No

$$\mathrm{y}''(t)=\mathrm{x}(t)-0.1\mathrm{y}'(t)+0.3\mathrm{y}(t)$$

$$\mathrm{y}''(t)+0.1\mathrm{y}'(t)-0.3\mathrm{y}(t)=\mathrm{x}(t)$$

The characteristic equation is

$$\lambda^2+0.1\lambda-0.3=0$$

The eigenvalues are $\lambda=-0.6$ and 0.5. The second eigenvalue has a positive real part. Therefore the system is unstable.

10. (3 pts) Any arbitrary CT signal convolved with the unit CT impulse, $\delta(t)$, is unchanged. That is, $\mathrm{x}(t) * \delta(t) = \mathrm{x}(t)$. What is the general mathematical relationship between $\mathrm{x}(t)$ and $\mathrm{x}(t) * \mathrm{u}(t)$?

$$\mathrm{x}(t) * \mathrm{u}(t) = \int_{-\infty}^{\infty} \mathrm{x}(\tau)\mathrm{u}(t-\tau)d\tau = \int_{-\infty}^{t} \mathrm{x}(\tau)d\tau$$

$\mathrm{x}(t) * \mathrm{u}(t)$ is the integral of $\mathrm{x}(t)$.