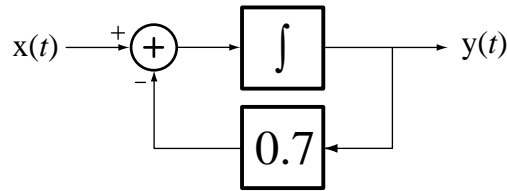


Solution of EE 503 Test #4 Su03 6/27/03

1. (6 pts) Find the transfer function, $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$ for the system illustrated below.

Which kind of ideal filter does this system approximate?

Filter Type is Lowpass



$$y'(t) = x(t) - 0.7y(t)$$

$$y'(t) + 0.7y(t) = x(t)$$

$$j\omega Y(s) + 0.7Y(s) = X(s)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{s + 0.7}$$

2. (2 pts) The highest frequency present in a signal is called the Nyquist frequency and the sampling rate that separates oversampling from undersampling is called the Nyquist rate.

3. (1 pt) Any signal sampled at exactly the Nyquist rate for all time can theoretically be reconstructed exactly from the samples.

False

4. (1 pt) A signal can be unlimited in both time and frequency simultaneously.

True

5. (1 pt) It is theoretically possible to exactly reconstruct any bandlimited, periodic signal for all time using a finite number of samples from the signal if the samples are taken in exactly the right way.

True

6. (1 pt) What is the most general functional description of a signal that, when used to excite an LTI system, produces a response of the same functional form?
complex exponential

7. (2 pts) Why is the time shifting property of the Laplace transform only valid for shifts to the right (delays) and not for shifts to the left?

If the signal is shifted to the left, it becomes non-causal and the unilateral Laplace transform does not apply.

8.

(a) (7 pts) Find the inverse Laplace transform, $h(t)$, of the transfer function,

$$H(s) = \frac{s^2}{(s+3)(s+5)}$$

$h(t) =$ _____

$$H(s) = \frac{s^2}{(s+3)(s+5)} = 1 - \frac{8s+15}{(s+3)(s+5)} = 1 - \left[\frac{-\frac{9}{2}}{s+3} + \frac{\frac{25}{2}}{s+5} \right]$$
$$h(t) = \delta(t) + \left[\frac{9}{2}e^{-3t} - \frac{25}{2}e^{-5t} \right] u(t)$$

(b) (1 pt) Which kind of ideal filter (lowpass, highpass, bandpass or bandstop) does this system transfer function represent? Filter type is highpass

(c) (2 pts) What is the region of convergence (ROC) in the s plane for this transfer function?

ROC is $\sigma > -3$.

9. Find the Nyquist rates for these signals. If a signal is not bandlimited, just write “infinite”.

(a) (5 pts) $x(t) = \text{rect}(t)(1 + \cos(2\pi t))$ Nyquist rate = infinite

(b) (5 pts) $x(t) = \cos(2\pi t)\sin(2\pi t)$ Nyquist rate = 4 Hz.

$$X(f) = \frac{1}{2}[\delta(f-1) + \delta(f+1)] * \frac{j}{2}[\delta(f+1) - \delta(f-1)]$$

$$X(f) = \frac{j}{4}[\delta(f) + \delta(f+2) - \delta(f-2) - \delta(f)] = \frac{j}{4}[\delta(f+2) - \delta(f-2)]$$

Highest frequency is 2 Hz. Nyquist rate is 4 Hz.

(c) (5 pts) $x(t) = \text{tri}(t) * \sin(2\pi t)$ Nyquist rate = 2 Hz.

$$X(f) = \text{sinc}^2(f) \frac{j}{2}[\delta(f+1) - \delta(f-1)] = \frac{j}{2}[\text{sinc}^2(-1)\delta(f+1) - \text{sinc}^2(+1)\delta(f-1)]$$

Highest frequency is 1 Hz. Nyquist rate is 2 Hz.

10. (4 pts) A signal, $x(t)$, is sampled 8 times to form the set of samples,

$$\{1, 8, -4, 4, 5, 2, a, -7\} .$$

The DFT, $X[k]$, of those 8 samples is

$$\{X[0], X[1], \dots, X[7]\} .$$

If $X[0] = 4$, what is the numerical value of a ? $a = \underline{-5}$

$X[0]$ is the sum of all the sample values. Therefore $4 = 1 + 8 - 4 + 4 + 5 + 2 + a - 7 \Rightarrow a = -5$

11. (7 pts) A signal, $x(t)$, is sampled 8 times to form the set of samples,

$$\{a, b, a, b, a, b, a, b\} .$$

The DFT, $X[k]$, of those 8 samples is

$$\{X[0], X[1], \dots, X[7]\} .$$

If $X[0] = 10$ and $X[4] = 0$, find the numerical values of a and b .

$$a = \underline{1.25}$$

$$b = \underline{1.25}$$

$$X[k] = \sum_{n=0}^{N_F-1} x[n] e^{-j2\pi \frac{kn}{N_F}} \Rightarrow X[0] = \sum_{n=0}^7 x[n] = 10 = 4a + 4b$$

$$X[k] = \sum_{n=0}^{N_F-1} x[n] e^{-j2\pi \frac{kn}{N_F}} \Rightarrow X[4] = \sum_{n=0}^7 x[n] (-1)^n = 0 = 4a - 4b \Rightarrow a = b$$

$$8a = 10 \Rightarrow a = b = 1.25$$