

# Solution of EE 503 Final Examination Su03

## 7/3/03

1. Let  $z$  represent any arbitrary complex number, and let  $w = z^{\frac{1}{n}}$ .
  - (a) Let  $n = 5$ . Then there are 5 unique values of  $w$ , the roots of the equation.
    - i. Is it possible that none of the roots,  $w$ , is a real number? Yes
    - ii. Is it possible that exactly one of the roots,  $w$ , is a real number? Yes
    - iii. Is it possible that exactly two of the roots,  $w$ , are real numbers? No
  - (b) Let  $n = 6$ . Then there are 6 unique values of  $w$ .
    - i. Is it possible that none of the roots,  $w$ , is a real number? Yes
    - ii. Is it possible that exactly one of the roots,  $w$ , is a real number? No
    - iii. Is it possible that exactly two of the roots,  $w$ , are real numbers? Yes
2. (1 pt) The angle (phase) of a product of two complex numbers is the sum of the angles (phases) of the two individual numbers.
3. (1 pt) The Cauchy-Riemann conditions are used to determine whether a function is analytic at a point.
4. (5 pts) Of the following functions, which are single-valued and which are multiple-valued?
  - (a)  $\sin(z)$  Single
  - (b)  $\sqrt{z}$  Multiple  
(Because of a common interpretation of the radical symbol, I counted both "Single" and "Multiple" as correct.)
  - (c)  $\log(z)$  Multiple
  - (d)  $e^z$  Single
  - (e)  $\tan^{-1}(z)$  Multiple
5. (1 pt) If a simple closed contour beginning and ending at a point,  $z = z_0$ , in the complex plane encircles a branch point of a function, the function value does not return to its initial value even though  $z$  does.

6. (1 pt) Which of the digital filter design techniques covered in the course can create an unstable DT system from a stable CT system? Finite difference
7. (1 pt) The sum of two periodic functions is always periodic. False
8. (1 pt) When a DT function is scaled by the transformation,  $n \rightarrow an$ , where “ $n$ ” is discrete time and “ $a$ ” is an integer greater than one, some of the values of the original function do not appear in the scaled function. This effect is called decimation.
9. (1 pt) If, in a system, unique excitations produce unique responses, the system is invertible.
10. (1 pt) If a system is both homogeneous and additive it is linear.
11. (11 pts) Find the impulse response,  $h[n]$ , of a DT system described by

$$40y[n] = 39y[n-1] + x[n]$$

where  $x$  is the excitation and  $y$  is the response. Also find the numerical value of  $h[30]$ .

$$h[n] = \frac{1}{40} \left( \frac{39}{40} \right)^n u[n]$$

$$h[30] = 0.011697$$

$$h[n] = 0, \quad n < 0, \quad h[n] = K_h \alpha^n, \quad n > 0, \quad 40\alpha^n = 39\alpha^{n-1} \Rightarrow \alpha = \frac{39}{40}$$

$$h[n] = K_h \left( \frac{39}{40} \right)^n, \quad n > 0, \quad h[0] = \frac{1}{40} \Rightarrow K_h = \frac{1}{40}, \quad h[n] = \frac{1}{40} \left( \frac{39}{40} \right)^n, \quad n \geq 0$$

$$h[n] = \frac{1}{40} \left( \frac{39}{40} \right)^n u[n], \quad h[30] = \frac{1}{40} \left( \frac{39}{40} \right)^{30} u[30] = 0.011697$$

12. If  $y(t) = \text{rect}(t-1) * \text{rect}(t-4)$  what are the numerical values of  $y(-5)$ ,  $y(0)$  and  $y(5)$ ?

$$y(-5) = \underline{0}$$

$$y(0) = \underline{0}$$

$$y(5) = \underline{1}$$

$$y(t) = \text{rect}(t-1) * \text{rect}(t-4) = \int_{-\infty}^{\infty} \text{rect}(\tau-1) \text{rect}(t-\tau-4) d\tau$$

$$y(-5) = \int_{-\infty}^{\infty} \text{rect}(\tau-1) \text{rect}(-\tau-9) d\tau = \int_{-\infty}^{\infty} \underbrace{\text{rect}(\tau-1) \text{rect}(\tau+9)}_{0 \text{ for all } \tau} d\tau = 0$$

$$y(0) = \int_{-\infty}^{\infty} \text{rect}(\tau-1) \text{rect}(-\tau-4) d\tau = \int_{-\infty}^{\infty} \underbrace{\text{rect}(\tau-1) \text{rect}(\tau+4)}_{0 \text{ for all } \tau} d\tau = 0$$

$$y(5) = \int_{-\infty}^{\infty} \text{rect}(\tau-1) \text{rect}(-\tau+1) d\tau = \int_{-\infty}^{\infty} \text{rect}(\tau-1) \text{rect}(\tau-1) d\tau$$

$$y(5) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \text{rect}(\tau-1) \text{rect}(\tau-1) d\tau = 1$$

Alternate Solution:

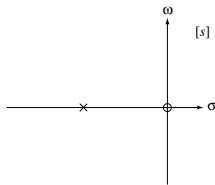
$$\text{rect}(t-1) * \text{rect}(t-4) \xrightarrow{\mathcal{F}} \text{sinc}(f) e^{-j2\pi f} \text{sinc}(f) e^{-j8\pi f} = \text{sinc}^2(f) e^{-j10\pi f}$$

$$y(t) = \text{tri}(t-5) \xrightarrow{\mathcal{F}} \text{sinc}^2(f) e^{-j10\pi f}$$

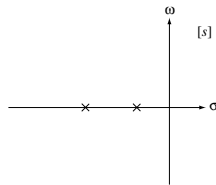
$$y(-5) = \text{tri}(-10) = \underline{0} \quad y(0) = \text{tri}(-5) = \underline{0} \quad y(5) = \text{tri}(0) = \underline{1}$$

13. (1 pt) A CT system is BIBO stable if its impulse response is absolutely integrable.

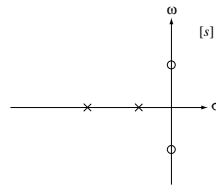
14. (8 pts) The four pole-zero plots below are for four filters. Identify for each pole-zero plot the type of filter, lowpass, highpass, bandpass, bandstop.



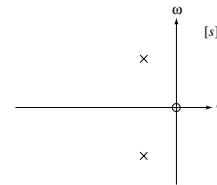
Highpass



Lowpass



Bandstop



Bandpass

15. (10 pts) Using the step-invariant technique and a sampling rate of 10 Hz, design a digital filter to approximate the CT filter whose transfer function is

$$H(s) = \frac{4s}{s+5} .$$

Write the  $z$ -domain transfer function,  $H(z)$ , as a ratio of polynomials in  $z$  with real coefficients.

$$H(s) = \frac{4s}{s+5} \Rightarrow H_{-1}(s) = \frac{4}{s+5} \Rightarrow h_{-1}(t) = 4e^{-5t} u(t) \Rightarrow h_{-1}[n] = 4e^{-5n(0.1)} u[n] = 4(0.6065)^n u[n]$$

$$h_{-1}[n] = 4(0.6065)^n u[n] \Rightarrow H_{-1}(z) = \frac{4z}{z-0.6065} \Rightarrow H(z) = \frac{z-1}{z} \frac{4z}{z-0.6065} = \frac{4(z-1)}{z-0.6065}$$

16. (3 pts) A signal,  $x(t) = 10\sin(50\pi t)$  is sampled at a rate of 20 Hz. This is undersampling. What signal, when properly sampled according to Shannon's sampling theorem, would yield exactly the same set of samples?

Subtract the sampling rate from the frequency.  $25 - 20 = 5$ . Therefore the signal is  $x(t) = 10\sin(10\pi t)$ , which is oversampled at a sampling rate of 20 Hz.