Solution to Test #1 ECE 504 F02

1. Five cards, two identical "x's", two identical "y's" and a z. Three positions. How many distinguishable arrangments?

1x, 1y, 1z: 3 things, 3 at a time, all distinguishable $N = \frac{3!}{0!} = 6$

xyz xzy yxz yzx zxy zyx

2x's and 1y: 3 things, 3 at a time, two indistiguishable "x's" $N = \frac{3!}{2! \times 0!} = 3$

xxy xyx yxx

2x's and 1z: 3 things, 3 at a time, two indistiguishable "x's" $N = \frac{3!}{2! \times 0!} = 3$

xxz xzx zxx

2y's and 1x: 3 things, 3 at a time, two indistiguishable "y's" $N = \frac{3!}{2! \times 0!} = 3$

yyx yxy xyy

2y's and 1z: 3 things, 3 at a time, two indistiguishable "y's" $N = \frac{3!}{2! \times 0!} = 3$

yyz yzy zyy

18 total distinguishable arrangements

2. $\mathrm{F}_X(x) = \left[1 - e^{-a(x - x_0)}\right]\mathrm{u}(x - x_0)$

$$\Pr(x_1 < x < x_2) = \mathrm{F}_X(x_2) - \mathrm{F}_X(x_1) = \left[1 - e^{-a(x_1 - x_0)}\right]\mathrm{u}(x_1 - x_0) - \left[1 - e^{-a(x_2 - x_0)}\right]\mathrm{u}(x_2 - x_0)$$

$$x_0 < x_1 < x_2$$

$$\Pr(x_1 < x < x_2) = e^{-a(x_2 - x_0)} - e^{-a(x_1 - x_0)}$$

3. A coin is flipped until a head appears or N_0 flips have occurred. The number of flips is N. What is $\mathrm{E}(N)$?

$$\Pr(N=1)=\frac{1}{2}\,,\ \Pr(N=2)=\frac{1}{4}\,,\ \cdots\Pr(N=N_0-1)=\frac{1}{2^{N_0-1}}$$

$$\Pr(N=N_0)=1-\left[\Pr(N=1)+\Pr(N=2)+\cdots+\Pr(N=N_0-1)\right]=\frac{1}{2^{N_0-1}}$$

$$\mathrm{E}(N)=\sum_{n=1}^{N_0} n\Pr(N=n)=\frac{N_0}{2^{N_0-1}}+\sum_{n=1}^{N_0-1}\frac{n}{2^n}$$

4. A man can go to the office by two routes. Route #1: N_1 minutes to a bridge, N_2 more minutes to the office. Route #2: N_3 minutes to the office. $N_3 > N_1 + N_2$. $\Pr(\text{bridge out}) = p_0$. When bridge is out he returns home and then takes the longer route. What is his expected travel time, $\mathrm{E}(\tau)$?

$$\mathrm{E}(\tau)=(N_1+N_2)(1-p_0)+(2N_1+N_3)p_0$$

5. A probability space is $\{A,B,C,D,E,F\}$. All outcomes are equally likely.

(a) Number of events is 2^N where N is the number of outcomes, in this case, 6. Number of events is 64.

(b) Event, X, is $\{A,C,F\}$ and event, Y, is $\{B,C,E\}$. $X \cup Y = \{A,B,C,E,F\}$.

$$\Pr(X\cup Y)=\Pr(A)+\Pr(B)+\Pr(C)+\Pr(E)+\Pr(F)=\frac{5}{6}$$

6. The pdf of X is constant between x_1 and x_2 , $x_2 > x_1$, and has an impulse of strength, 0.4, at x_3.

$$\mathrm{E}(X)=\int_{-\infty}^{\infty} x\,\mathrm{p}_X(x)dx = K\int_{x_1}^{x_2} xdx + 0.4x_3$$

$$K=\frac{0.6}{x_2-x_1}$$

$$\mathrm{E}(X)=\frac{0.6}{x_2-x_1}\int_{x_1}^{x_2} xdx+0.4x_3=\frac{0.6}{x_2-x_1}\frac{1}{2}\left(x_2^2-x_1^2\right)+0.4x_3=0.6\frac{x_2+x_1}{2}+0.4x_3$$

7. Two men and two women. Probability of a man being left-handed is 0.15. Probability of a woman being left-handed is 0.1.

$$\Pr(2\text{ LH people}) = \Pr(0\text{ LHM})\Pr(2\text{ LHW}) + \Pr(1\text{ LHM})\Pr(1\text{ LHW}) + \Pr(2\text{ LHM})\Pr(0\text{ LHW})$$

$$\Pr(0\text{ LHM})\Pr(2\text{ LHW}) = (0.85)^2(0.1)^2 = 0.007225$$

$$\Pr(1\text{ LHM})\Pr(1\text{ LHW}) = \binom{2}{1}(0.15)^1(0.85)^1\binom{2}{1}(0.1)^1(0.9)^1$$

$$\Pr(1\text{ LHM})\Pr(1\text{ LHW}) = 4\left[(0.15)(0.85)(0.1)(0.9)\right] = 0.0459$$

$$\Pr(2\text{ LHM})\Pr(0\text{ LHW}) = (0.15)^2(0.9)^2 = 0.018225$$

$$\Pr(2\text{ LH people}) = 0.007225 + 0.0459 + 0.018225 = 0.07135$$