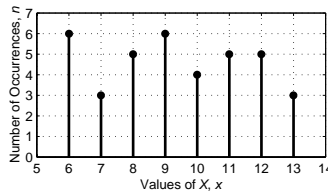


# Solution to Test #3 in ECE 504

1. A random noise process has a power spectral density which is a constant,  $K$ , between  $f = -f_m$  to  $f = f_m$  and zero elsewhere. A 50% duty-cycle square wave oscillating between  $A$  and  $B$  is added to a sample function of this random process to form,  $Z(t)$ . What is the signal power (mean-squared value) of  $Z(t)$  ?

The two signals are independent. That means that their autocorrelation functions and therefore their mean-squared values simply add. The mean-squared value of the random noise is the product,  $2Kf_m$  (the area under its PSD). The mean-squared value of the square wave is  $\frac{(A^2 + B^2)}{2}$ . Therefore the signal power of  $Z(t)$  is  $2Kf_m + \frac{(A^2 + B^2)}{2}$ .

2. A random variable,  $X$ , has the sample histogram below. (a) Based on this histogram what is the best estimate of the probability of the value, 9 ? (b) What is the best estimate of the mean value of  $X$ ?



The best estimate of the probability of 9 is the relative frequency of 9 which is the number of times it occurs divided by the total number of occurrences of all values or

$$\Pr(X = 9) = \frac{6}{37} = 0.1622$$

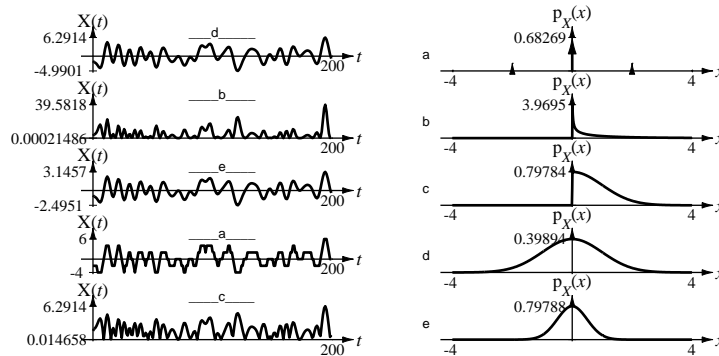
The best estimate of the expected value of  $X$  is the sample mean which is

$$\bar{X} = \frac{1}{N} \sum_{i=1}^M n_i x_i = \frac{36 + 21 + 40 + 54 + 40 + 55 + 60 + 39}{37} = 9.32$$

3. A Gaussian-distributed random variable,  $X$ , has a mean of 20 and a standard deviation of 8. An estimate is made of its mean value by averaging a sample of 291 randomly-chosen values. What is the confidence level that the population mean is within a range of values from 0.23448 below the sample mean to 0.23448 above the sample mean?

The standard deviation of the sample mean is  $\sigma_{\bar{x}} = \frac{8}{\sqrt{291}} = 0.46896$ . The value, 0.23448 is one-half of this standard deviation of the sample mean. Since the original population was Gaussian distributed, the sample mean is also. Therefore the confidence level is the probability that a normally-distributed random variable is between  $-\frac{1}{2}$  and  $\frac{1}{2}$ . Consulting a table of the normal distribution, that probability (and confidence level) is approximately 0.382925 or 38.2925%.

4. Match the graphs of sample functions to the pdf's by writing the letter of the correct pdf in the space provided with each sample function.



5. An ergodic random process,  $X$ , has a mean of  $-0.43265$  and a standard deviation of  $1.6799$ . A non-stationary random process,  $Y$ , is formed by multiplying  $X$  by zero for times less than zero and by time,  $t$ , for times greater than zero. What are the expected value and variance of  $Y$  as a function of time over all time ?

$$Y(t) = tX(t)u(t)$$

$$E(Y(t)) = E(tX(t)u(t)) = tE(X(t))u(t) = -0.43265tu(t)$$

$$\sigma_Y^2 = E(Y^2(t)) - [E(Y(t))]^2$$

$$E(Y^2(t)) = E(t^2 X^2(t)u^2(t)) = t^2 E(X^2(t))u(t) = t^2 [2.822 + (-0.43265)^2]u(t)$$

$$[E(Y(t))]^2 = (-0.43265tu(t))^2 = 0.43265^2 t^2 u(t)$$

$$\sigma_Y^2 = t^2 [2.822 + (-0.43265)^2]u(t) - 0.43265^2 t^2 u(t) = 2.822t^2 u(t)$$

6. A random process has sample functions consisting of adjacent rectangular pulses of width,  $w$ . Each pulse may have a value of  $A$  or  $B$  with equal probability and the value of any pulse does not depend on the value of any other pulse. What are (a) the expected value of the random process, (b) the mean-squared value of this random process and (c) the value of the autocorrelation function for this random process at a delay,  $\tau$ , of exactly one pulse width,  $R_x(w)$  ?

(a) The expected value is  $E(X) = \Pr(A)A + \Pr(B)B = \frac{1}{2}A + \frac{1}{2}B = \frac{A+B}{2}$ .

(b) The mean-squared value is  $E(X^2) = \Pr(A)A^2 + \Pr(B)B^2 = \frac{1}{2}A^2 + \frac{1}{2}B^2 = \frac{A^2+B^2}{2}$ .

(c) The autocorrelation at a delay of exactly one pulse width is

$$R_x(w) = \Pr(A-A)A^2 + \Pr(A-B)AB + \Pr(B-A)BA + \Pr(B-B)B^2$$

$$R_x(w) = \Pr(A)\Pr(A)A^2 + \Pr(A)\Pr(B)AB + \Pr(B)\Pr(A)BA + \Pr(B)\Pr(B)B^2$$

$$R_x(w) = \frac{1}{2} \times \frac{1}{2}A^2 + \frac{1}{2} \times \frac{1}{2}AB + \frac{1}{2} \times \frac{1}{2}BA + \frac{1}{2} \times \frac{1}{2}B^2 = \frac{A^2 + B^2 + 2AB}{4}$$

$$R_x(w) = \left(\frac{A+B}{2}\right)^2 = (E(X))^2$$