Solution of ECE 505 Final Examination F06

1. Two signals $x_1 = \{3,8,-2,5\}$ and $x_2 = \{-1,-4,3,0\}$ are circularly convolved to form the signal x. The DFT of x is X. Find the numerical values of X.

$X(0)$	$X(1)$	$X(2)$	$X(3)$
-28	$-8+j32$	-72	$-8-j32$

$$X_1(0) = 3+8-2+5 = 14 \qquad X_2(0) = -1-4+3+0 = -2$$
$$X_1(1) = 3-j8+2+j5 = 5-j3 \qquad X_2(1) = -1+j4-3+0 = -4+j4$$
$$X_1(2) = 3-8-2-5 = -12 \qquad X_2(2) = -1+4+3+0 = 6$$
$$X_1(3) = 3+j8+2-j5 = 5+j3 \qquad X_2(3) = -1-j4-3+0 = -4-j4$$

$$X = X_1 X_2 = \{14\times(-2),(5-j3)(-4+j4),-12\times 6,(5+j3)(-4-j4)\}$$
$$X = \{-28,-8+j32,-72,-8-j32\}$$

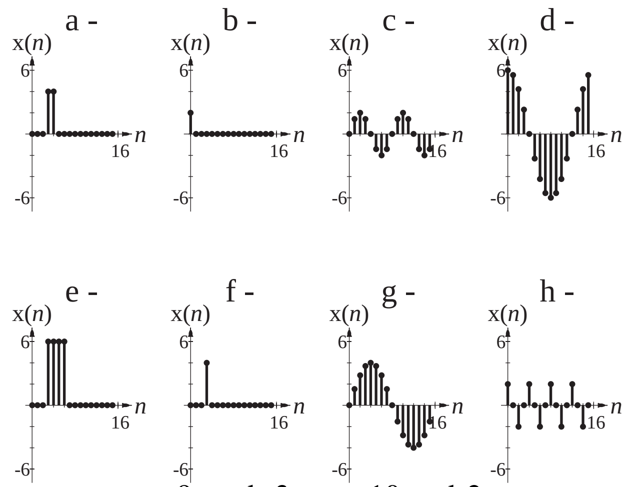
Alternate Solution:

$$x = x_1 \circledast x_2$$
$$x(0) = \{3,8,-2,5\}\times\{-1,0,3,-4\} = -3+0-6-20 = -29$$
$$x(1) = \{3,8,-2,5\}\times\{-4,-1,0,3\} = -12-8+0+15 = -5$$
$$x(2) = \{3,8,-2,5\}\times\{3,-4,-1,0\} = 9-32+2+0 = -21$$
$$x(3) = \{3,8,-2,5\}\times\{0,3,-4,-1\} = 0+24+8-5 = 27$$

$$x = \{-29,-5,-21,27\}$$
$$X = \{-28,-8+j32,-72,-8-j32\}$$

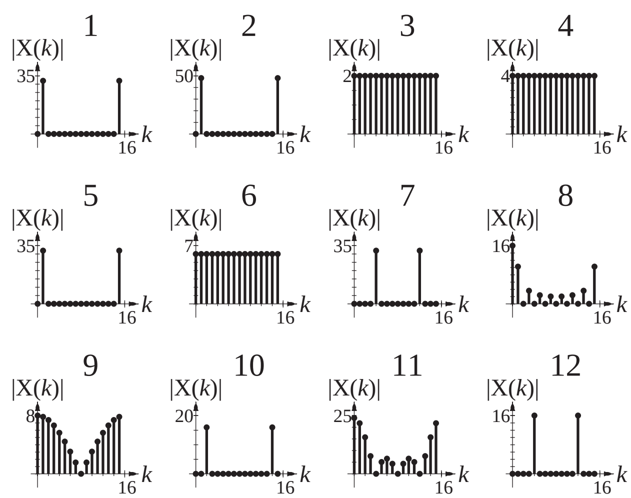
Check.

2. For each function of *n* find the corresponding graph of the magnitude of its DFT as a function of *k*. Write the number designation of the *k* function right beside the letter designation of the *n* function (after the dash -).



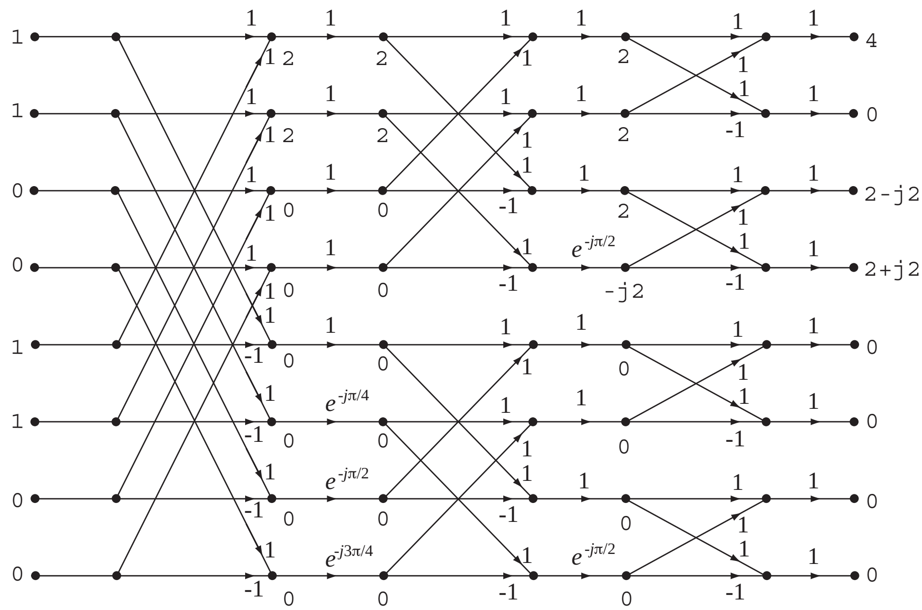
a-9 b-3 c-10 d-2

e-11 f-4 g-1 or 5 h-12



3. Using the fft diagram below find the DFT of the sequence $x = \{1,1,0,0,1,1,0,0\}$. Write in all numerical values directly on the diagram in the boxes provided. Fill in the numerical values of the DFT (X) in the spaces below.

X(0)	X(1)	X(2)	X(3)
4	0	$2-j2$	0
X(4)	**X(5)**	**X(6)**	**X(7)**
0	0	$2+j2$	0



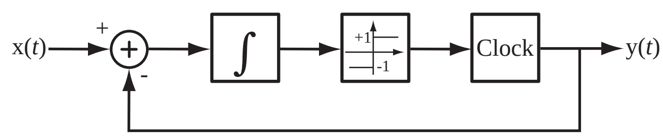
4. In the sigma-delta converter below the comparator executes the signum function. That is, if the input signal to the comparator is $x_c(t)$ and the output signal from the comparator is $y_c(t)$ then

$$y_c(t) = \begin{cases} 1 , & x_c(t) > 0 \\ 0 , & x_c(t) = 0 \\ -1 , & x_c(t) < 0 \end{cases} .$$

The clock samples the output of the comparator once per second and that value is held at the output of the clock until the next sample is taken. The output signal from the integrator $y_i(t)$ is the integral of its input signal $x_i(t)$ with no other gains or scale factors.

$$y_i(t) = \int_{-\infty}^{t} x_i(\tau)\,d\tau$$

The initial state of the output signal is $y(t) = 0$. The clock starts operating at time $t = 0$. The input signal is $x(t) = 0.3\,u(t)$. Fill in the table below with numerical values. (The notation n^+ means $\lim_{\varepsilon \to 0}(n+\varepsilon)$, $\varepsilon > 0$.)



t	0^+	1^+	2^+	3^+	4^+	5^+	6^+	7^+	8^+	9^+
$x(t) - y(t)$	0.3	–0.7	1.3	–0.7	–0.7	1.3	–0.7	–0.7	1.3	–0.7
$y_i(t)$	0	0.3	–0.4	0.9	0.2	–0.5	0.8	0.1	–0.6	0.7
$y(t)$	0	1	–1	1	1	–1	1	1	–1	1

5. A digital filter has a transfer function $\mathrm{H}(z) = K\dfrac{z^2 - 2z\cos(\pi/3) + 1}{z^2 - 2rz\cos(\pi/3) + r^2}$. If $r = 0.8$ and $K = 4$ and the input signal to the filter is $\mathrm{x}(n) = 10\cos(2\pi n/12)\mathrm{u}(n)$, what function does the output signal $\mathrm{y}(n)$ approach as $n \to \infty$?

$$\mathrm{H}(z) = 4\frac{z^2 - 2z\cos(\omega_0) + 1}{z^2 - 1.6z\cos(\omega_0) + 0.64}$$

$$\mathrm{H}\left(e^{j\pi/6}\right) = 4\frac{\left(e^{j\pi/6}\right)^2 - 2e^{j\pi/6}\cos(\pi/3) + 1}{\left(e^{j\pi/6}\right)^2 - 1.6e^{j\pi/6}\cos(\pi/3) + 0.64} = 4.5336e^{-0.2824}$$

$$\lim_{n\to\infty} \mathrm{y}(n) = 10\left|\mathrm{H}(\pi/6)\right|\cos\left(2\pi n/12 + \measuredangle\mathrm{H}(\pi/6)\right)$$

$$\lim_{n\to\infty} \mathrm{y}(n) = 45.336\cos(2\pi n/12 - 0.2824)$$