Solution of ECE 505 Test 1 F06

1. A signal $\mathrm{x}(t) = 4\cos(200\pi t) + 7\sin(340\pi t)$ is sampled by an A/D converter at a rate of 80 samples per second. These samples are then input to a D/A converter operating at the same rate (converting samples to analog voltage 80 times per second). The output signal from the D/A converter is then filtered by an ideal filter which removes all signal power above half the sampling rate (40 Hz). Let the output signal from the filter be $\mathrm{y}(t)$.

(a) What numerical frequencies are present in $\mathrm{y}(t)$?

Frequencies present ____

$$\mathrm{x}(t) = 4\cos(200\pi t) + 7\sin(340\pi t) \Rightarrow \mathrm{x}(n) = 4\cos(200\pi n/80) + 7\sin(340\pi n/80)$$

$$\mathrm{x}(n) = 4\cos(5\pi n/2) + 7\sin(17\pi n/4)$$

$$\mathrm{x}(n) = 4\cos(5\pi n/2 - 4\pi n/2) + 7\sin(17\pi n/4 - 16\pi n/4) = 4\cos(\pi n/2) + 7\sin(\pi n/4)$$

$$\mathrm{y}(t) = 4\cos(\pi 80t/2) + 7\sin(\pi 80t/4) = 4\cos(40\pi t) + 7\sin(20\pi t)$$

20 Hz and 10 Hz are present

(b) What is the numerical value of $\mathrm{y}(0.00625)$?

$\mathrm{y}(0.00625) =$ ____

$$\mathrm{y}(0.00625) = 4\cos(40\pi \times 0.00625) + 7\sin(20\pi \times 0.00625) = 5.507$$

2. A discrete-time system has an input-output relationship

$$\mathrm{y}(n) = \sum_{m=-\infty}^{n+2} \mathrm{x}(m) \; .$$

Circle correct answers and explain your answer. An answer without an explanation gets no credit.

(a) Is it linear? [Linear] Non-Linear

Let $\mathrm{x}_1(n) = \mathrm{g}(n)$. Then $\mathrm{y}_1(n) = \sum_{m=-\infty}^{n+2} \mathrm{g}(m)$.

Let $\mathrm{x}_2(n) = \mathrm{h}(n)$. Then $\mathrm{y}_2(n) = \sum_{m=-\infty}^{n+2} \mathrm{h}(m)$.

Let $\mathrm{x}_3(n) = \alpha\mathrm{g}(n) + \beta\mathrm{h}(n)$. Then

$$\mathrm{y}_3(n) = \sum_{m=-\infty}^{n+2} \alpha\mathrm{g}(m) + \beta\mathrm{h}(m) = \alpha \sum_{m=-\infty}^{n+2} \mathrm{g}(m) + \beta \sum_{m=-\infty}^{n+2} \mathrm{h}(m) = \alpha\mathrm{y}_1(n) + \beta\mathrm{y}_2(n)$$

Linear

(b) Is is time invariant? [Time Invariant] Time Variant

Let $x_1(n) = g(n)$. Then $y_1(n) = \sum_{m=-\infty}^{n+2} g(m)$.

Let $x_2(n) = g(n-n_0)$. Then $y_2(n) = \sum_{m=-\infty}^{n+2} g(m-n_0) = \sum_{k=-\infty}^{n+2-n_0} g(k) = y_1(n-n_0)$.

Time Invariant.

(c) Is it stable? Stable **[Unstable]**

If $x(n)$ is a constant, y is unbounded. Therefore it is unstable.

(d) Is it causal? Causal **[Non-Causal]**

Present value of y depends on future values of x. Non-causal.

3. Write the sequence representing the correlation of these two sequences. Be sure to indicate the zero position in time ($n = 0$) .

$$\left\{-1, 3, \underset{\uparrow}{2}\right\} \qquad \left\{2, \underset{\uparrow}{0}, 4\right\}$$

At zero shift

$$\begin{array}{cccc} -1 & 3 & 2 & 0 \\ 0 & 2 & 0 & 4 \\ 0 & 6 & 0 & 0 & \Rightarrow 6 \end{array}$$

At a shift of one,

$$\begin{array}{ccccc} -1 & 3 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 4 \\ 0 & 0 & 4 & 0 & 0 & \Rightarrow 4 \end{array}$$

At a shift of more than one to the right the result is zero.

At a shift of minus one,

$$\begin{array}{ccc} -1 & 3 & 2 \\ 2 & 0 & 4 \\ -2 & 0 & 8 & \Rightarrow 6 \end{array}$$

At a shift of minus two,

$$\begin{array}{cccc} 0 & -1 & 3 & 2 \\ 2 & 0 & 4 & 0 \\ 0 & 0 & 12 & 0 & \Rightarrow 12 \end{array}$$

At a shift of minus three,

$$\begin{array}{ccccc} 0 & 0 & -1 & 3 & 2 \\ 2 & 0 & 4 & 0 & 0 \\ 0 & 0 & -4 & 0 & 0 & \Rightarrow -4 \end{array}$$

At a shift of more than three to the left, the result is zero.

So the correlation sequence is $\left\{-4, 12, 6, \underset{\uparrow}{6}, 4\right\}$.

4. If the system function of a system is

$$\mathrm{H}(z) = \frac{z+4}{(z+0.2)(z-3)} = \frac{z^{-1}+4z^{-2}}{\left(1+0.2z^{-1}\right)\left(1-3z^{-1}\right)},$$

indicate the region of convergence (ROC) that makes this system stable and find the impulse response of the system $\mathrm{h}(n)$.

For the system to be stable the ROC must contain the unit circle. Therefore it must lie between two circles of radius 0.2 and radius 3.

$$\mathrm{H}(z) = \frac{z+4}{(z+0.2)(z-3)} = \frac{-1.1875}{z+0.2} + \frac{2.1875}{z-3} \quad , \quad 0.2 < |z| < 3$$

$$\mathrm{H}(z) = \frac{z^{-1}+4z^{-2}}{\left(1+0.2z^{-1}\right)\left(1-3z^{-1}\right)} = \frac{-1.1875z^{-1}}{1+0.2z^{-1}} + \frac{2.1875z^{-1}}{1-3z^{-1}} \quad , \quad 0.2 < |z| < 3$$

$$\mathrm{h}(n) = -1.1875(-0.2)^{n-1}\,\mathrm{u}(n-1) - 2.1875(3)^{n-1}\,\mathrm{u}(-n)$$

Solution of ECE 505 Test 1 F06

1. A signal $\mathrm{x}(t) = 4\cos(140\pi t) + 7\sin(280\pi t)$ is sampled by an A/D converter at a rate of 90 samples per second. These samples are then input to a D/A converter operating at the same rate (converting samples to analog voltage 90 times per second). The output signal from the D/A converter is then filtered by an ideal filter which removes all signal power above half the sampling rate (45 Hz). Let the output signal from the filter be $\mathrm{y}(t)$.

(a) What numerical frequencies are present in $\mathrm{y}(t)$?

Frequencies present ____

$$\mathrm{x}(t) = 4\cos(140\pi t) + 7\sin(280\pi t) \Rightarrow \mathrm{x}(n) = 4\cos(140\pi n/90) + 7\sin(280\pi n/90)$$

$$\mathrm{x}(n) = 4\cos(14\pi n/9) + 7\sin(28\pi n/9)$$

$$\mathrm{x}(n) = 4\cos(14\pi n/9 - 18\pi n/9) + 7\sin(28\pi n/9 - 36\pi n/9)$$

$$= 4\cos(-4\pi n/9) + 7\sin(-8\pi n/9)$$

$$\mathrm{x}(n) = 4\cos(4\pi n/9) - 7\sin(8\pi n/9)$$

$$\mathrm{y}(t) = 4\cos(4\pi 90t/9) - 7\sin(8\pi 90t/9) = 4\cos(40\pi t) - 7\sin(80\pi t)$$

20 Hz and 40 Hz are present

(b) What is the numerical value of $\mathrm{y}(0.00625)$?

$\mathrm{y}(0.00625) =$ ____

$$\mathrm{y}(0.00625) = 4\cos(40\pi \times 0.00625) - 7\sin(80\pi \times 0.00625) = -4.1716$$

2. A discrete-time system has an input-output relationship

$$\mathrm{y}(n) = \sum_{m=-\infty}^{n-2} \mathrm{x}(m) \; .$$

Circle correct answers and explain your answer. An answer without an explanation gets no credit.

(a) Is it linear? **[Linear]** Non-Linear

Let $\mathrm{x}_1(n) = \mathrm{g}(n)$. Then $\mathrm{y}_1(n) = \sum_{m=-\infty}^{n-2} \mathrm{g}(m)$.

Let $\mathrm{x}_2(n) = \mathrm{h}(n)$. Then $\mathrm{y}_2(n) = \sum_{m=-\infty}^{n-2} \mathrm{h}(m)$.

Let $\mathrm{x}_3(n) = \alpha\,\mathrm{g}(n) + \beta\,\mathrm{h}(n)$. Then

$$y_3(n) = \sum_{m=-\infty}^{n-2} \alpha g(m) + \beta h(m) = \alpha \sum_{m=-\infty}^{n-2} g(m) + \beta \sum_{m=-\infty}^{n-2} h(m) = \alpha y_1(n) + \beta y_2(n)$$

Linear

(b) Is is time invariant? **[Time Invariant]** Time Variant

Let $x_1(n) = g(n)$. Then $y_1(n) = \sum_{m=-\infty}^{n-2} g(m)$.

Let $x_2(n) = g(n-n_0)$. Then $y_2(n) = \sum_{m=-\infty}^{n-2} g(m-n_0) = \sum_{k=-\infty}^{n-2-n_0} g(k) = y_1(n-n_0)$.

Time Invariant.

(c) Is it stable? Stable **[Unstable]**

If $x(n)$ is a constant, y is unbounded. Therefore it is unstable.

(d) Is it causal? **[Causal]** Non-Causal

Present value of y depends only on past values of x. Causal.

3. Write the sequence representing the correlation of these two sequences. Be sure to indicate the zero position in time ($n = 0$) .

$$\left\{-1, 3, \underset{\uparrow}{2}\right\} \qquad \left\{4, \underset{\uparrow}{0}, 2\right\}$$

At zero shift

$$\begin{matrix} -1 & 3 & 2 & 0 & \\ 0 & 4 & 0 & 2 & \\ 0 & 12 & 0 & 0 & \Rightarrow 12 \end{matrix}$$

At a shift of one,

$$\begin{matrix} -1 & 3 & 2 & 0 & 0 & \\ 0 & 0 & 4 & 0 & 2 & \\ 0 & 0 & 8 & 0 & 0 & \Rightarrow 8 \end{matrix}$$

At a shift of more than one to the right the result is zero.

At a shift of minus one,

$$\begin{matrix} -1 & 3 & 2 & \\ 4 & 0 & 2 & \\ -4 & 0 & 4 & \Rightarrow 0 \end{matrix}$$

At a shift of minus two,

$$\begin{matrix} 0 & -1 & 3 & 2 & \\ 4 & 0 & 2 & 0 & \\ 0 & 0 & 6 & 0 & \Rightarrow 6 \end{matrix}$$

At a shift of minus three,

$$\begin{matrix} 0 & 0 & -1 & 3 & 2 & \\ 4 & 0 & 2 & 0 & 0 & \\ 0 & 0 & -2 & 0 & 0 & \Rightarrow -2 \end{matrix}$$

At a shift of more than three to the left, the result is zero.

So the correlation sequence is $\left\{-2, 6, 0, \underset{\uparrow}{12}, 8\right\}$.

4. If the system function of a system is

$$\mathrm{H}(z) = \frac{z+3}{(z+0.5)(z-2)} = \frac{z^{-1}+3z^{-2}}{\left(1+0.5z^{-1}\right)\left(1-2z^{-1}\right)},$$

indicate the region of convergence (ROC) that makes this system stable and find the impulse response of the system $\mathrm{h}(n)$.

For the system to be stable the ROC must contain the unit circle. Therefore it must lie between two circles of radius 0.2 and radius 3.

$$\mathrm{H}(z) = \frac{z+3}{(z+0.5)(z-2)} = \frac{-1}{z+0.5} + \frac{2}{z-2} \quad , \quad 0.5 < |z| < 2$$

$$\mathrm{H}(z) = \frac{z^{-1}+3z^{-2}}{\left(1+0.5z^{-1}\right)\left(1-2z^{-1}\right)} = \frac{-z^{-1}}{1+0.5z^{-1}} + \frac{2z^{-1}}{1-2z^{-1}} \quad , \quad 0.5 < |z| < 2$$

$$\mathrm{h}(n) = -(-0.5)^{n-1}\,\mathrm{u}(n-1) - 2(2)^{n-1}\,\mathrm{u}(-n)$$