Solution of ECE 505 Test 2 F06

1. A continuous-time sinusoidal signal $\mathrm{x}_a(t) = 5\cos(2000\pi t)$ is sampled at $f_s = 1/T = 5000$ samples/second to form the sinusoidal discrete-time signal $\mathrm{x}(n) = \mathrm{x}_a(nT)$. The signal $\mathrm{x}(n)$ is applied to a discrete-time system with transfer function $\mathrm{H}(z) = \dfrac{0.7z}{z-0.3} = \dfrac{0.7}{1-0.3z^{-1}}$. The output signal from the system is also sinusoidal of the form $\mathrm{y}(n) = A\cos(\omega_0 n + \theta)$. Find the numerical values of A, ω_0 and θ.

$$\mathrm{x}(n) = 5\cos(2000\pi nT) = 5\cos(2\pi n/5)$$

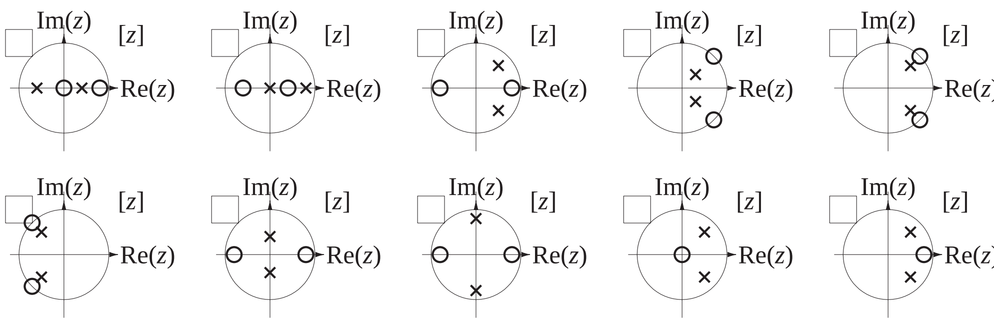
$$\mathrm{H}\left(e^{j2\pi/5}\right) = \frac{0.7e^{j2\pi/5}}{e^{j2\pi/5} - 0.3} = 0.7021 - j0.2208 = 0.736\measuredangle - 0.3047 \text{ or } \measuredangle\text{-}17.46°$$

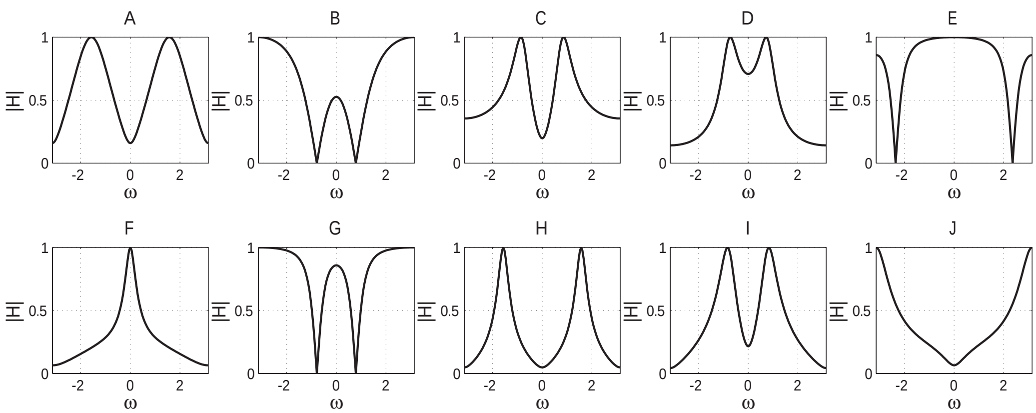
So the response amplitude is $5 \times 0.736 = 3.68$ and the response phase shift relative to the excitation is -0.3047 radians and

$$\mathrm{y}(n) = 3.68\cos\left((2\pi/5)n - 0.3047\right).$$

2. In the boxes provided, write the letter of the frequency response magnitude graph that corresponds to each pole-zero plot.

J	F	I	B	G
E	A	H	D	C





3. A signal $x_a(t)$ has a continuous-time Fourier transform for which

$$\begin{aligned} X_a(F) &\neq 0 \ , \ 200 < |F| < 230 \\ X_a(F) &= 0 \ , \ \text{otherwise} \end{aligned}.$$

What is the numerical minimum sampling rate for which the signal can be recovered exactly from the samples?

$$k_{\max} = \left\lfloor \frac{F_{\mathrm{H}}}{B} \right\rfloor = \left\lfloor \frac{230}{230-200} \right\rfloor = 7$$

$$F_s = \frac{2F_{\mathrm{H}}}{k_{\max}} = \frac{460}{7} = 65.714$$

4. An 8-bit ADC with a full range of -10 V to +10V samples and quantizes a sinusoidal signal with an amplitude of 7 V. Using the usual assumptions about the distribution and power spectral density of the quantization noise, find the numerical signal-to-quantization-noise ratio (SQNR) of the ADC output signal (defined as the signal power of a signal with no quantization noise divided by the signal power of the quantization noise) in dB.

$$P_s = \frac{7^2}{2} = 24.5$$

$$\Delta = \frac{20}{2^8} = 0.078125 \Rightarrow P_q = \frac{\Delta^2}{12} = 0.000509$$

$$\text{SQNR}_{\text{dB}} = 10\log_{10}\left(P_s / P_q\right) = 46.82 \text{ dB}$$

Solution of ECE 505 Test 2 F06

1. A continuous-time sinusoidal signal $x_a(t) = 5\cos(2000\pi t)$ is sampled at $f_s = 1/T = 6000$ samples/second to form the sinusoidal discrete-time signal $x(n) = x_a(nT)$. The signal $x(n)$ is applied to a discrete-time system with transfer function $H(z) = \frac{0.7z}{z-0.3} = \frac{0.7}{1-0.3z^{-1}}$. The output signal from the system is also sinusoidal of the form $y(n) = A\cos(\omega_0 n + \theta)$. Find the numerical values of A, ω_0 and θ.

$$x(n) = 5\cos(2000\pi nT) = 5\cos(\pi n/3)$$

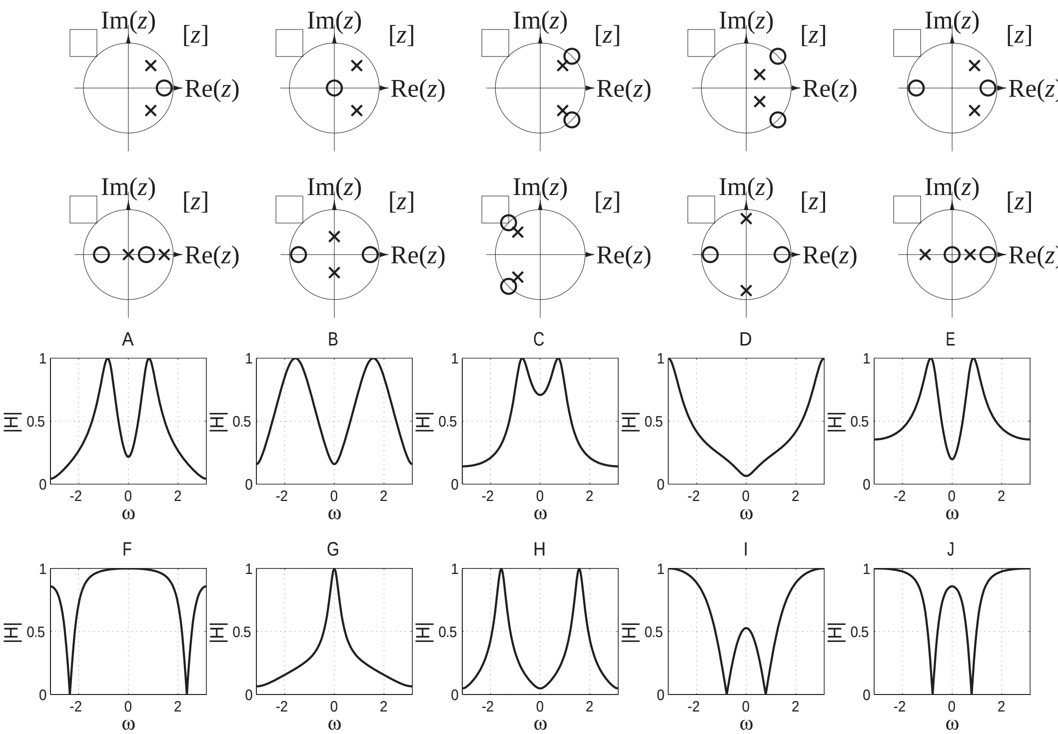
$$H(e^{j\pi/3}) = \frac{0.7e^{j\pi/3}}{e^{j\pi/3} - 0.3} = 0.7532 - j0.2302 = 0.7876\measuredangle - 0.2966 \text{ or } \measuredangle\text{-}16.996^\circ$$

So the response amplitude is $5 \times 0.736 = 3.938$ and the response phase shift relative to the excitation is -0.2966 radians and

$$y(n) = 3.938\cos((\pi/3)n - 0.2966).$$

2. In the boxes provided, write the letter of the frequency response magnitude graph that corresponds to each pole-zero plot.

E	C	J	I	A
G	B	F	H	D



3. A signal $x_a(t)$ has a continuous-time Fourier transform for which

$$\begin{aligned} X_a(F) &\neq 0 \ , \ 210 < |F| < 230 \\ X_a(F) &= 0 \ , \ \text{otherwise} \end{aligned} .$$

What is the numerical minimum sampling rate for which the signal can be recovered exactly from the samples?

$$k_{\max} = \left\lfloor \frac{F_{\mathrm{H}}}{B} \right\rfloor = \left\lfloor \frac{230}{230-210} \right\rfloor = 11$$

$$F_s = \frac{2F_{\mathrm{H}}}{k_{\max}} = \frac{460}{11} = 41.82$$

4. An 8-bit ADC with a full range of -10 V to +10V samples and quantizes a sinusoidal signal with an amplitude of 8 V. Using the usual assumptions about the distribution and power spectral density of the quantization noise, find the numerical signal-to-quantization-noise ratio (SQNR) of the ADC output signal (defined as the signal power of a signal with no quantization noise divided by the signal power of the quantization noise) in dB.

$$P_s = \frac{8^2}{2} = 32$$

$$\Delta = \frac{20}{2^8} = 0.078125 \Rightarrow P_q = \frac{\Delta^2}{12} = 0.000509$$

$$\text{SQNR}_{\text{dB}} = 10\log_{10}\left(P_s / P_q\right) = 47.98 \text{ dB}$$

$$\delta[n] \xleftrightarrow{\mathcal{Z}} 1 \quad , \quad \text{All } z$$

$$\alpha^n \, \mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \frac{z}{z-\alpha} = \frac{1}{1-\alpha z^{-1}} \quad , \quad |z| > |\alpha|$$

$$n \, \mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \frac{z}{(z-1)^2} = \frac{z^{-1}}{(1-z^{-1})^2} \quad , \quad |z| > 1$$

$$\alpha^n \sin(\Omega_0 n) \mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \frac{z\alpha \sin(\Omega_0)}{z^2 - 2\alpha z \cos(\Omega_0) + \alpha^2} = \frac{\alpha \sin(\Omega_0) z^{-1}}{1 - 2\alpha \cos(\Omega_0) z^{-1} + \alpha^2 z^{-2}} \quad , \quad |z| > |\alpha|$$

$$\alpha^n \cos(\Omega_0 n) \mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \frac{z[z - \alpha \cos(\Omega_0)]}{z^2 - 2\alpha z \cos(\Omega_0) + \alpha^2} = \frac{1 - \alpha \cos(\Omega_0) z^{-1}}{1 - 2\alpha \cos(\Omega_0) z^{-1} + \alpha^2 z^{-2}} \quad , \quad |z| > |\alpha|$$

$$-\alpha^n \, \mathrm{u}[-n-1] \xleftrightarrow{\mathcal{Z}} \frac{z}{z-\alpha} = \frac{1}{1-\alpha z^{-1}} \quad , \quad |z| < |\alpha|$$

$$-n\alpha^n \, \mathrm{u}[-n-1] \xleftrightarrow{\mathcal{Z}} \frac{\alpha z}{(z-\alpha)^2} = \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2} \quad , \quad |z| < |\alpha|$$