## Sequence alignment (continued)

## Various types



## Global Alignment (review)

- Input: Two strings, labeled $s$ and $t$
$-|s|$ is $n$
$-|t|$ is $m$
- Output: Two strings $s_{A}$ and $t_{A}$ such that:
$-s_{A}$ and $t_{A}$ are of equal length $L$
- Characters must be in same order, with "-" spacers as needed
- If $s_{A}[]=$ ' $-’$, then $t_{A}[I] \neq{ }^{\prime}$ '
- If $t_{A}[I]={ }^{\prime}-$, then $s_{A}[I] \neq{ }^{\prime}$ '


## Visual alignments

Score: A measure of alignment quality


Total $=33$

Scored as $E(C, C) E(A,-), E(T, T), E(-, C)$, etc.

## Types of scores

- Reasons:
- Sequencing error(s)
- Evolutionary change
- Three parameters
- Gap (indel?)
- Mismatch (misread base?)
- Match (no change)
- We will only consider constant gap penalties for now.


## Optimal alignment

- We want to find an alignment that optimizes an evaluation function $E\left(s_{A}, t_{A}\right)$
- The "brute force" method is a hard problem (NP-hard) requiring exponential time.
- Dynamic programming, in cases we will discuss, can be used to solve this problem efficiently.


## Requirements

- We will need four things to compute a global alignment:

1. Substitution matrix (parameters)
2. Recurrence relation
3. Filling up a table
4. Traceback

We can solve this based on looking at three smaller problems

$$
T[i, j]=\max \left\{\begin{array}{l}
T[i-1, j-1]+\operatorname{score}(s[i], t[j]) \\
T[i-1, j]+g \\
T[i, j-1]+g
\end{array}\right.
$$

## ACTACTA GACTAC



|  | $\lambda$ | C | T | C | G | C | A | G | C |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\lambda$ |  |  |  | -15 | -20 | -25 | $-30$ | -35 | -40 |
| C |  |  | $\geq 5$ |  |  |  |  |  |  |
| A | -10 |  |  |  |  |  |  |  |  |
| T | -15 |  |  |  |  |  |  |  |  |
| T | -20 |  |  |  |  |  |  |  |  |
| C | -25 |  |  |  |  |  |  |  |  |
| A | -30 |  |  |  |  |  |  |  |  |
| C | -35 |  |  |  |  |  |  |  |  |

+10 for match, -2 for mismatch, -5 for space (rowwise)

|  | $\lambda$ | C | T | C | G | C | A | G | C |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | -5 | -10 | -15 | -20 | -25 | -30 | -35 | -40 |
|  | -5 | 10 | 5 | 0 | -5 | -10 | -15 | -20 | -25 |
|  | -10 | 5 | 8 | 3 | -2 | -7 | 0 | -5 | -10 |
| T | -15 | 0 | 15 | 10 | 5 | 0 | -5 | -2 | -7 |
| T | -20 | -5 | 10 | 13 | 8 | 3 | -2 | -7 | -4 |
| C | -25 | -10 | 5 | 20 | 15 | 18 | 13 | 8 | 3 |
| A | -30 | -15 | 0 | 15 | 18 | 13 | 28 | 23 | 18 |
| C | -35 | -20 | -5 | 10 | 13 | 28 | 23 | 26 33 |  |

Traceback yields both optimal alignments in this example

## End-gap free alignment

- We often don't want to penalize gaps at the start or end of the alignment, especially when comparing short and long sequences
- Same as global alignment, except:
- Initialize with zeros (free gaps at start)
- Locate max in the last row/column (free gaps at end)
- Also called semiglobal alignment


## Example paths



## Example traceback paths

|  | Sequence B AGCTAAGCTAA |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\checkmark$ | G-2 | 2 |  |  |  |  |  |  |  |  |  |  |  |
|  | $\mathrm{C}^{-3}$ | 3 |  |  | 3 |  |  |  |  |  |  |  |  |
|  | C- | $4-1$ | 12 |  | , |  |  |  |  |  |  |  |  |
| E | T | -5-2 | 21 | 4 | - |  |  |  |  |  | 6 |  | 5 |
| d | A | $6-3$ | 30 | 3 |  |  |  |  |  |  |  |  | - |
| ป | A | 7 | 4-1 |  |  |  |  |  |  |  |  |  |  |
| $\sigma$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\pm$ | C | $8-5$ | $5^{-2}$ |  | . |  |  |  |  |  |  |  |  |
| $\sim$ | C | - | $6^{-3}$ | 0 |  |  |  |  |  |  |  |  |  |
|  | A | -10-7 |  | -1 |  |  |  |  |  |  |  |  |  |
|  |  | 12-8 |  |  |  |  |  |  |  |  |  |  |  |

$$
\begin{aligned}
& \text {-AGCC-TAG-CCAA } \\
& \text { AGCTAAG-CT-A-A } \\
& \text { AGCCTA-GCCAA } \\
& \text { AGCT-AAGCTAA } \\
& \text { AGCCTA-GCCAA } \\
& \text { AGC-TAAGCTAA } \\
& \text {-----------AGCCTAGCCAA } \\
& \text { AGCTAAGCTAA---------- }
\end{aligned}
$$

## Practice

$$
\begin{gathered}
\mathrm{T}[i, 0]=0 \quad \mathrm{~T}[0, j]=0 \\
T[i, j]=\max \begin{cases}T[i-1, j-1]+\operatorname{score}(s[i], t[j]) \\
T[i-1, j]+g & \text { Match }=5 \\
T[i, j-1]+g & \text { Mismatch }=-2\end{cases} \\
\begin{array}{ll}
\mathrm{S}=\text { CATTA } & \\
\mathrm{T}=\text { ACATTAG } & \text { Find the croro of }
\end{array}
\end{gathered}
$$

Find the score of the best semiglobal alignment
$\begin{array}{lllllllll}\lambda & \mathrm{C} & \mathrm{T} & \mathrm{C} & \mathrm{G} & \mathrm{C} & \mathrm{A} & \mathrm{G} & \mathrm{C}\end{array}$

|  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |

+10 for match, -2 for mismatch, -5 for space (rowwise)

## Local Alignment

## $T[i, j]=$ Score of optimally aligning a suffix of $s$ with a suffix of $t$.

$$
T[i, j]=\max \left\{\begin{array}{l}
T[i-1, j-1]+\operatorname{score}(s[i], t[j]) \\
T[i-1, j]+g \\
T[i, j-1]+g \\
0
\end{array}\right.
$$

Initialize top row and leftmost column to zero.

## Practice

$$
\begin{gathered}
\mathrm{T}[i, 0]=0 \quad \mathrm{~T}[0, j]=0 \\
T[i, j]=\max \begin{cases}T[i-1, j-1]+\operatorname{score}(s[i], t[j]) \\
T[i-1, j]+g & \text { Match }=5 \\
T[i, j-1]+g & \text { Mismatch }=-2 \\
0 & \text { Gap }=-3\end{cases} \\
\mathrm{S}=\text { CTACT } \\
\mathrm{T}=\text { ATACG }
\end{gathered}
$$

Find the score of the best local alignment

| $\lambda$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| A | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 |
| T | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| T | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 1 | 0 | 2 | 0 | 1 | 0 | 0 | 1 |
| A | 0 | 0 | 0 | 0 | 1 | 0 | 2 | 0 | 0 |
| C | 0 | 1 | 0 | 1 | 0 | 2 | 0 | 1 | 1 |

## Reducing space requirements

- O (mn) tables are often the limiting factor in computing large alignments
- There is a linear space technique that only doubles the time required [Hirschberg77]


## Alignment and new architectures

- http://en.wikipedia.org/wiki/SmithWaterman_algorithm

