Dynamic Response of an Active Filter Using a Generalized Nonactive Power Theory

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***Abstract*—This paper presents a theory of instantaneous nonactive power/current. This generalized theory is independent of the number of phases, whether the load is periodic or non-periodic, and whether the system voltages are balanced or unbalanced. By choosing appropriate parameters such as the averaging interval T_c and the reference voltage v_p, the theory has different forms for each specific system application. This theory is consistent with other more traditional concepts. The theory is implemented in a parallel nonactive power compensation system, and several different cases, such as harmonics load, rectifier load, single-phase pulse load, and non-periodic load, are simulated in MATLAB. Unity power factor or pure sinusoidal source current from the utility can be achieved according to different compensation requirements. Furthermore, the dynamic response and its impact on the compensator's energy storage requirement are also presented.**

***Keywords* — nonactive power, reactive power, non-periodic current , shunt compensator, STATCOM.**

I. INTRODUCTION

Nonlinear loads draw highly distorted currents from the utility as well as cause distortion of the voltages. Some loads introduce harmonics into the system; some draw irregular currents which are not periodic; some have subharmonics whose frequencies are not integer multiples of the fundamental frequency; some are single-phase loads or unbalanced which cause asymmetry in a three-phase four-wire system. Variable-speed motor drives, arc furnaces, computer power supplies, and unbalanced single-phase loads are the most common nonlinear loads in power systems.

A transient disturbance may also be considered as one kind of non-periodic current from the compensation point of view. The disturbance may be caused by the sudden change of a large load such as starting a motor, a fault, or a sudden load change. The compensator is often required to have larger power rating, larger energy storage, and faster switching frequency if it is responsible to mitigate such disturbances.

Time-based instantaneous nonactive power theory was first formulated in the 1930's by Fryze [1], and the increasing rate of nonlinear loads requires a more comprehensive theory to describe, measure, and compensate these loads. However, most of the previous efforts have focused on periodic non-sinusoidal systems, rather than non-periodic systems [2]. The *p-q* theory proposed in [3] is valid for three-phase three-wire systems with harmonics, and it has been extended to three-phase four-wire systems [4-5]. In [6], Hilbert space techniques are adopted for the definition and compensation of reactive power. However, it introduces new harmonics to the active current which is not desired in nonactive power compensation. Non-integer multiple harmonics are defined as non-periodic currents in [7] and the compensation is discussed. Non-periodic currents are also discussed in [8].

The diversity of the features of non-periodic currents makes it difficult to get one definition that fits all situations, and the compensation of such currents is quite difficult. However, from a practical point of view, regardless of the load, a fundamental sinusoidal source current in phase with the system voltage is usually the preferred objective of compensation.

The theory proposed in this paper is a generalized one which is independent of the number of phases in the system, applies whether the voltages are balanced or unbalanced, and to both periodic and non-periodic loads. Applied to a parallel (shunt) active filter, this theory provides flexibility in nonactive power compensation. By changing the averaging interval T_c and the reference voltage v_p, the active filter can compensate the nonactive component in the load current so that the source current will be fundamental sinusoidal, or have the same waveform as the system voltage, or any desired waveform, whether the system voltage is distorted or not.

Combined with the direct current control scheme, both steady state and dynamic response are studied. It is shown that an active filter with the proposed nonactive power theory has a fast dynamic response which is within one fundamental cycle. Several factors related to the definition itself and the implementation are discussed. These factors include the averaging interval T_C, the coupling inductance L_C, the DC link voltage v_{dc}, and the capacitance requirement C of the active filter. They are determined by the characteristics of the load, the rating limits of the active filter, and the desired compensation results.

II. INSTANTANEOUS NONACTIVE POWER THEORY

The theory was first presented in [9]. The voltage vector $\boldsymbol{v}(t)$ and current vector $\boldsymbol{i}(t)$ are defined as, respectively,

$$\boldsymbol{v}(t) \triangleq [v_1, v_2, \ldots, v_m]^T, \tag{1}$$

$$\boldsymbol{i}(t) \triangleq [i_1, i_2, ..., i_m]^T, \tag{2}$$

where m is the number of phases. The instantaneous active current $\boldsymbol{i_a}(t)$ and the instantaneous nonactive current $\boldsymbol{i_n}(t)$, respectively, are defined as

$$\boldsymbol{i_a}(t) \triangleq \frac{P(t)}{V_p^2(t)} \boldsymbol{v_p}(t), \tag{3}$$

$$\boldsymbol{i_n}(t) \triangleq \boldsymbol{i}(t) - \boldsymbol{i_a}(t). \tag{4}$$

where $P(t)$ is the average value of the instantaneous power $p(t)$ over the averaging interval $[t\text{-}T_c, t]$:

$$P(t) \triangleq \frac{1}{T_c} \int_{t-T_c}^{t} p(\tau) d\tau = \frac{1}{T_c} \int_{t-T_c}^{t} \boldsymbol{v}(\tau)^T \boldsymbol{i}(\tau) d\tau. \tag{5}$$

Let $\boldsymbol{v_p}(t)$ denote the reference voltage, which can be the source voltage $\boldsymbol{v}(t)$ itself, or some other reference such as the fundamental component of $\boldsymbol{v}(t)$. $V_p(t)$ is the rms value of the reference voltage $\boldsymbol{v_p}(t)$, given by

$$V_p(t) \triangleq \sqrt{\frac{1}{T_c} \int_{t-T_c}^{t} v_p(\tau)^T v_p(\tau) d\tau}. \tag{6}$$

Based on the definitions for $\boldsymbol{i_a}$(t) and $\boldsymbol{i_n}(t)$, the instantaneous active power $p_a(t)$ and instantaneous nonactive power $p_n(t)$ are defined as, respectively,

$$p_a(t) \triangleq \boldsymbol{v}(t)^T \boldsymbol{i_a}(t) \tag{7}$$

$$p_n(t) \triangleq \boldsymbol{v}(t)^T \boldsymbol{i_n}(t). \tag{8}$$

where $p_a(t)$ and $p_n(t)$ satisfy,

$$p(t) = p_a(t) + p_n(t). \tag{9}$$

$I(t)$, $I_a(t)$, and $I_n(t)$ are the rms values of $\boldsymbol{i}(t)$, $\boldsymbol{i_a}(t)$, and $\boldsymbol{i_n}(t)$, respectively. $\boldsymbol{i_a}(t)$ and $\boldsymbol{i_n}(t)$ are orthogonal so that,

$$I^2(t) = I_a^2(t) + I_n^2(t). \tag{10}$$

For a periodic system with period T, and choosing $T_c = T/2$, and $\boldsymbol{v_p}(t) = \boldsymbol{v}(t)$, the average active power and nonactive power satisfy

$$P_a(t) = \frac{1}{T_c} \int_{t-T_c}^{t} \boldsymbol{v}(\tau)^T \boldsymbol{i_a}(\tau) d\tau \ \equiv P(t), \tag{11}$$

$$P_n(t) = \frac{1}{T_c} \int_{t-T_c}^{t} \boldsymbol{v}(\tau)^T \boldsymbol{i_n}(\tau) d\tau \ \equiv 0. \tag{12}$$

Over the time interval T_c, $P_a(t)$, the average value of $p_a(t)$, is equal to $P(t)$, the average value of $p(t)$; and $P_n(t)$, the average value of $p_n(t)$, is zero. It indicates that instantaneously $p(t)$ has both active and nonactive components, but on average, $P(t)$ has only the active power, which is equal to $P_a(t)$. The average value of $p_n(t)$ is zero, which indicates that the nonactive power $p_n(t)$ flows back and forth, and over the time interval T_c, there is no energy utilization. This is consistent with the conventional definition of active power and nonactive power. By injecting the same amount of current as the instantaneous nonactive current $\boldsymbol{i_n}$(t) into the system, a compensator can completely compensate the nonactive component in the load current without any energy source.

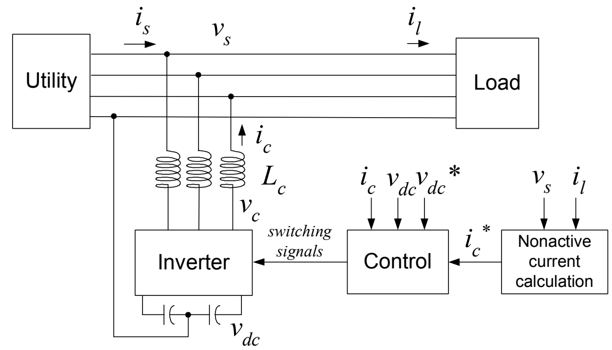


Fig.1. System configuration of shunt nonactive power compensation.

Unlike many other nonactive power/current theories, this theory does not have any limitations on the voltage $\boldsymbol{v}(t)$ and current $\boldsymbol{i}(t)$. It is a general definition independent of the number of phases, whether the voltage is sinusoidal or non-sinusoidal, and whether the load is periodic or non-periodic. More specifically, the definition is applicable to single-phase, three-phase three-wire, and three-phase four-wire systems. For each different case, it is consistent with the traditional definitions and takes on a specific form, by varying the reference voltage $\boldsymbol{v_p}(t)$ and the averaging interval T_c.

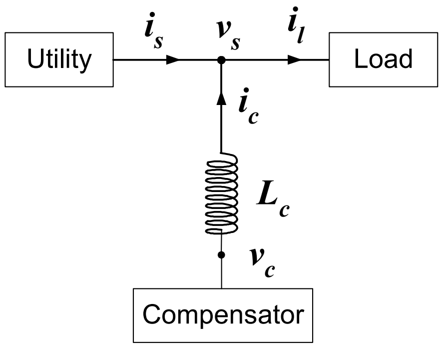
The reference voltage $\boldsymbol{v_p}(t)$ determines the shape of the instantaneous active current waveform, as shown in (3). If $P(t)$ and $V_p(t)$ are constant, which can be achieved with $T_c = T/2$ for a periodic system with period T, $\boldsymbol{i_a}(t)$ has exactly the same waveform as $\boldsymbol{v_p}(t)$. In practice, $\boldsymbol{v_p}(t)$ is chosen based on the voltage $\boldsymbol{v}(t)$, the current $\boldsymbol{i}(t)$, and the desired active current $\boldsymbol{i_a}(t)$. By choosing different reference voltages, $\boldsymbol{i_a}(t)$ can be reshaped so that the unwanted components in $\boldsymbol{i}(t)$ are eliminated; furthermore, the elimination of each harmonic component is independent of each other.

Theoretically, T_c could be chosen arbitrarily from zero to infinity. Practically, for a non-periodic system, T_c is chosen as a finite number based on the compensation objectives. For a periodic system with period T, T_c is chosen to be one half or a full period of T, i.e., $T_c = T/2$ or T, which will completely compensate the nonactive component in the load current.

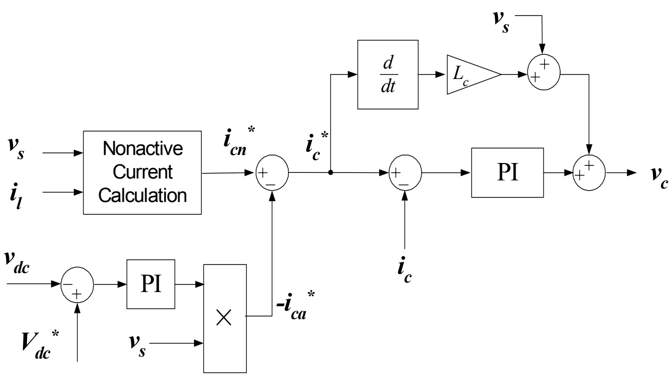
III. Implementation in a STATCOM

As previously mentioned, the theory is applicable to three-phase three-wire, three-phase four-wire, and single-phase systems. A three-phase four-wire parallel compensation system is shown in Fig. 1. The source current i_s is the sum of the load current i_l and the compensator current i_c. The goal of compensation is to make the compensator supply the nonactive component of the load current, i.e., $\boldsymbol{i_c}(t) = \boldsymbol{i_n}(t)$, so that the utility only provides the active current, i.e., $\boldsymbol{i_s}(t) = \boldsymbol{i_a}(t)$.

In the proposed theory, the compensation system is assumed to be lossless, however in a real system, losses exist in the switches, the capacitors and the inductors. Therefore a certain amount of active power is drawn to regulate DC link capacitor voltage. For a shunt active filter, a voltage source



(a) Equivalent circuit of the compensation system



(b) Control diagram of the shunt compensation system

Fig. 2 Control scheme of the compensation system.

inverter with a capacitor is mostly used. There are two components in the compensator current $\boldsymbol{i}_c$, the nonactive component $\boldsymbol{i}_{cn}$ to compensate the load current and the active component $\boldsymbol{i}_{ca}$ to meet the compensator's losses by regulating the DC link voltage v_{dc}, and

$$\boldsymbol{i}_c = \boldsymbol{i}_{cn} + \boldsymbol{i}_{ca}\,. \tag{13}$$

A PI controller is used to regulate the DC link voltage v_{dc}, as shown in Fig. 2b [10]. The active current needed to meet the losses is in phase with $\boldsymbol{v}_s$. The amplitude of the active current is controlled by the difference between the reference voltage V_{dc}^* and the actual DC link voltage v_{dc}. $\boldsymbol{i}_{ca}^*$ is in phase with $\boldsymbol{v}_s$ and is calculated as follows

$$\boldsymbol{i}_{ca}^* = -\boldsymbol{v}_s\left[K_{P1}(V_{dc}^* - v_{dc}) + K_{I1}\int_0^t (V_{dc}^* - v_{dc})dt\right], \tag{14}$$

where K_{P1} and K_{I1} are the proportional and integral gains of the PI controller.

The equivalent circuit of the shunt compensation system is shown in Fig. 2a, where $\boldsymbol{v}_s$ is the system voltage, $\boldsymbol{v}_c$ is the output voltage of the inverter, L_c is the coupling inductance, and $\boldsymbol{i}_c$ is the compensator current.

$$L_c\frac{d\boldsymbol{i}_c}{dt} = \boldsymbol{v}_c - \boldsymbol{v}_s \tag{15}$$

The active current of the compensator $\boldsymbol{i}_{ca}$ is a small fraction of the whole compensator current $\boldsymbol{i}_c$, so the active component can be neglected, i.e., $\boldsymbol{i}_{cn}^* \approx \boldsymbol{i}_c^*$, and $\boldsymbol{i}_{cn} \approx \boldsymbol{i}_c$, where $\boldsymbol{i}_{cn}^*$ is the nonactive current calculated based on the nonactive power/current theory discussed in Section II. The reference of the compensator nonactive current $\boldsymbol{i}_{cn}^*$ and the reference of the compensator output voltage $\boldsymbol{v}_c^*$ satisfy

$$L_c\frac{d\boldsymbol{i}_c^*}{dt} = \boldsymbol{v}_c^* - \boldsymbol{v}_s \tag{16}$$

That is, the reference compensator output voltage $\boldsymbol{v}_c^*$ is

$$\boldsymbol{v}_c^* = L_c\frac{d\boldsymbol{i}_c^*}{dt} + \boldsymbol{v}_s\,. \tag{17}$$

Subtract (15) from (16),

$$L_c\frac{d(\boldsymbol{i}_c^* - \boldsymbol{i}_c)}{dt} = \boldsymbol{v}_c^* - \boldsymbol{v}_c\,. \tag{18}$$

Set the control input $\boldsymbol{v}_c$ according to

$$\boldsymbol{v}_c = \boldsymbol{v}_s + L_c\frac{d\boldsymbol{i}_c^*}{dt} + L_c\left[K_{P2}(\boldsymbol{i}_c^* - \boldsymbol{i}_c) + K_{I2}\int_0^t (\boldsymbol{i}_c^* - \boldsymbol{i}_c)dt\right] \tag{19}$$

where K_{P2} and K_{I2} are the proportional and integral gains of the PI controller.

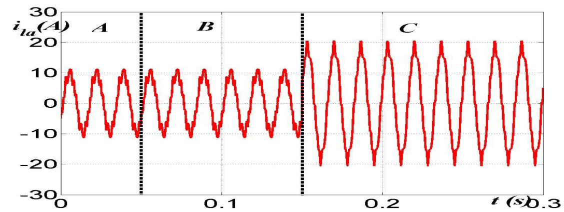
Fig. 2b is the complete control diagram of the shunt compensation system. The inner loop is the compensator current control which controls the output voltage of the compensator according to the required nonactive current $\boldsymbol{i}_c^*$. The outer loop controls the active current drawn by the compensator by regulating the DC link voltage v_{dc}, and combines the active current $\boldsymbol{i}_{ca}^*$ together with the nonactive current $\boldsymbol{i}_{cn}^*$.

IV. Case Studies of Dynamic Response

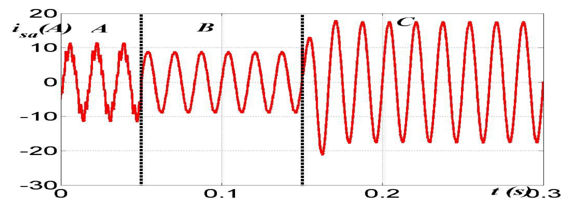
A Simulink model has been set up based on the diagram in Fig. 1. The parameters of the system are shown in Table I. Some parameters, such as the DC link voltage v_{dc}, the DC link capacitance C, and the coupling inductance, vary according to different compensation requirements. To illustrate the generalized characteristics of the proposed theory, four nonlinear loads are simulated, and the simulation results are analyzed. A sudden load change is applied to each case to study the dynamic response of the compensation system.

TABLE I. PARAMETERS OF THE STATCOM

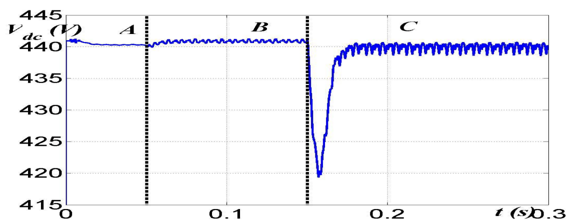
System voltage, line-to-line rms (V)	208
DC link voltage V_{dc} (V)	440.9
DC link capacitance (μF)	50-2000
Coupling inductance (mH)	10
Switching frequency (Hz)	20,000



(a) load current of phase a, $i_{la}(t)$



(b) source current of phase a, $i_{sa}(t)$



(c) DC link voltage v_{dc}

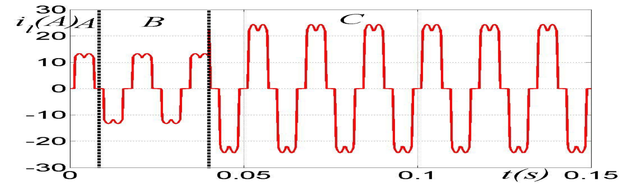
Fig. 3. Simulation of harmonic load compensation.

A. Three-phase Harmonic Load

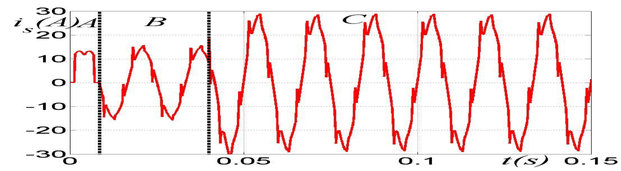
This is the most common nonlinear load in power systems. In this case, the load current has harmonics as shown in Fig. 3a, and for clarity, only phase a is shown.

For compensation of periodic currents with a fundamental period T, choosing different T_c alone does not change the source current characteristics. With reference to (3) and (4), the rms value of a periodic quantity does *not* depend on the time averaging interval T_c if it is an integer multiple of $T/2$. So with $T_c = T/2$, the source current after compensation is shown in Fig. 3b. In region A, the compensator is off, so that the source current is equal to the load current, which contains harmonics and fundamental nonactive power. In region B and region C, the compensator is working and the nonactive component in the load current is completely compensated, and the source current now is a fundamental sinusoid and in phase with the fundamental component of system voltage.

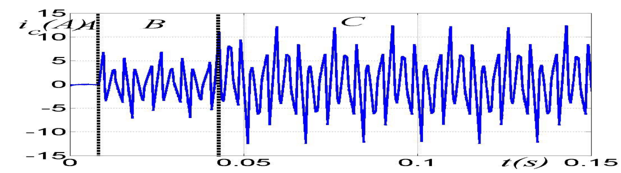
The load current has a sudden change at t = 0.15 s. The nonactive component of the increase is provided by the compensator, and the active component of the increase is provided by the utility, which is shown in region C, Fig. 3b. The sudden increase of output power causes a voltage drop on the DC link capacitor as shown in Fig. 3c. Compared to the steady-state operation, the DC link voltage variation is much larger, which demands higher capacitance to prevent the voltage from dropping to a low level. In Fig. 3c, the voltage variation in region C is larger than the one in region B, because after the load change, a larger amount of nonactive current is required from the compensator.



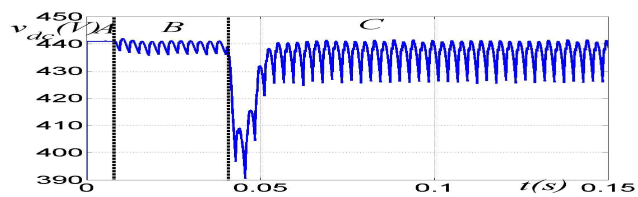
(a) load current of phase a, $i_{la}(t)$



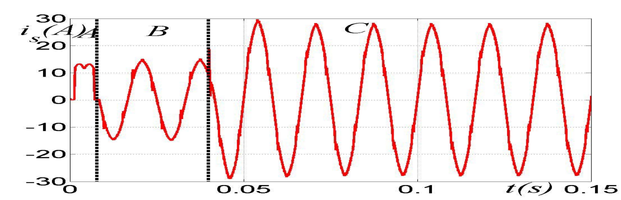
(b) source current of phase a, $i_{sa}(t)$, $V_{dc} = 1.5V_{ll}$



(c) compensator current of phase a, $i_{ca}(t)$, $V_{dc} = 1.5V_{ll}$



(d) DC link voltage v_{dc}, $V_{dc} = 1.5V_{ll}$



(e) source current of phase a, $i_{sa}(t)$, $V_{dc} = 3V_{ll}$

Fig. 4. Simulation of rectifier load compensation.

B. Three-phase Diode Rectifier Load

A three-phase diode rectifier is also a typical nonlinear load. The current of a diode rectifier is shown in Fig. 4a (phase a). It is a periodic waveform with the same period as the fundamental period of the system voltage, thus the current can be decomposed into a fundamental component and harmonics. However, different from the harmonics load discussed in the previous subsection, the current drawn by a diode-based rectifier is not continuous, and the variation of the current with time di/dt is very large ($di/dt = 5.8\times10^4$ A/s in this example). Therefore, there are special issues in the compensation of a rectifier load.

At t = 0.04 s, there is a sudden load change which causes the load current to increase (the second dash line in Fig. 4a). In Fig. 4b, region A shows $i_s(t)$ supplying the full load current when the compensator is off; when the compensator is on, region B shows that $i_s(t)$ is close to a sinusoid, and region C is $i_s(t)$ after the sudden load change. Fig. 4c shows the nonactive current which is provided by the compensator, which is highly distorted. The load change has an impact on the DC link voltage, which drops about 50 volts when the load changes. To regulate v_{dc}, some active current is drawn. The sudden load change requires a larger capacitance than a steady load.

The source current $i_{sa}(t)$ in Fig. 4b has some spikes, which is caused by the large variation of the current with time. According to (15), the compensator output voltage v_c is given by

$$v_c = v_s + L_c \frac{di_c}{dt}.$$

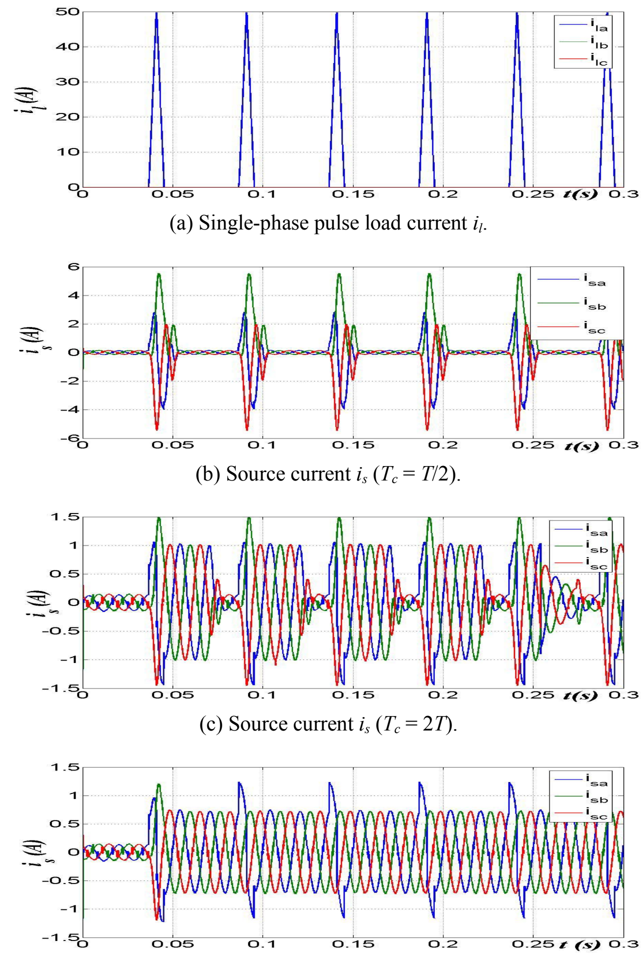
A large di_c/dt requires a large v_c; however, if the DC link voltage does not meet this requirement, the compensator is unable to provide the nonactive current as required by the nonactive power theory, and spikes occur in the source current accordingly. Fig. 4b and Fig. 4e show the source current when the DC link voltage V_{dc} is $1.5V_{ll}$ and $3V_{ll}$, respectively, where V_{ll} is the peak value of the system line-to-line voltage. The spikes in Fig. 4e are smaller than those in Fig. 4b, which shows that a source current with less spikes can be achieved by increasing the DC link voltage. However, a higher DC link voltage requires a higher voltage rating of the compensator and results in higher compensator losses. In practice, the DC link voltage can be chosen between V_{ll} to $2V_{ll}$.

C. Single-phase Pulse Load

If a single-phase load is connected to the three-phase utility, the single-phase current drawn from the system causes an unbalance of the system. Non-sinusoidal load current, or individual high amplitude pulse load current also introduces high nonactive components to the system. In this case, the load current is a single-phase pulse current in phase *a*, with a period of $3T$ (T is the fundamental period of the voltage). The pulse load current is shown in Fig. 5a. The voltage is a three-phase fundamental sine wave.

After compensation, the utility source current has currents in all the three phases, even though the load current is only in phase *a*. They are in phase with the system voltage, and the amplitudes are much smaller than the peak value of the load current, as shown in Fig. 5b. The small amount of current between two pulses is the active current drawn to meet the compensator's losses.

Unlike the previous two cases, the averaging interval T_c is a critical factor to the compensation in this case. Figs. 5b, 5c, and 5d show the three-phase source currents after compensation where T_c is $T/2$, $2T$, and $3T$, respectively. With a longer T_c, 1) i_s is closer to a sine wave; 2) the three phases are more balanced; 3) i_s is always in phase with the voltage; and 4) the magnitude of i_s is decreasing. The overshoots in the currents are because the actual compensator current does not exactly track the reference nonactive current, which is a sharp triangle in this case, and the DC link voltage control draws active current to meet the losses and to recover the large voltage drop as well. However, by increasing T_c, the compensator current i_c also increases, and consequently, the capacitance rating and switching current rating must increase as well. Depending on the load characteristics, compensator requirements, and the compensation results desired, a T_c can usually be chosen to meet all the objectives. In this case for example, $T_c = 2T$ has a good source current without too high of a compensator rating.



(a) Single-phase pulse load current i_l.

(b) Source current i_s ($T_c = T/2$).

(c) Source current i_s ($T_c = 2T$).

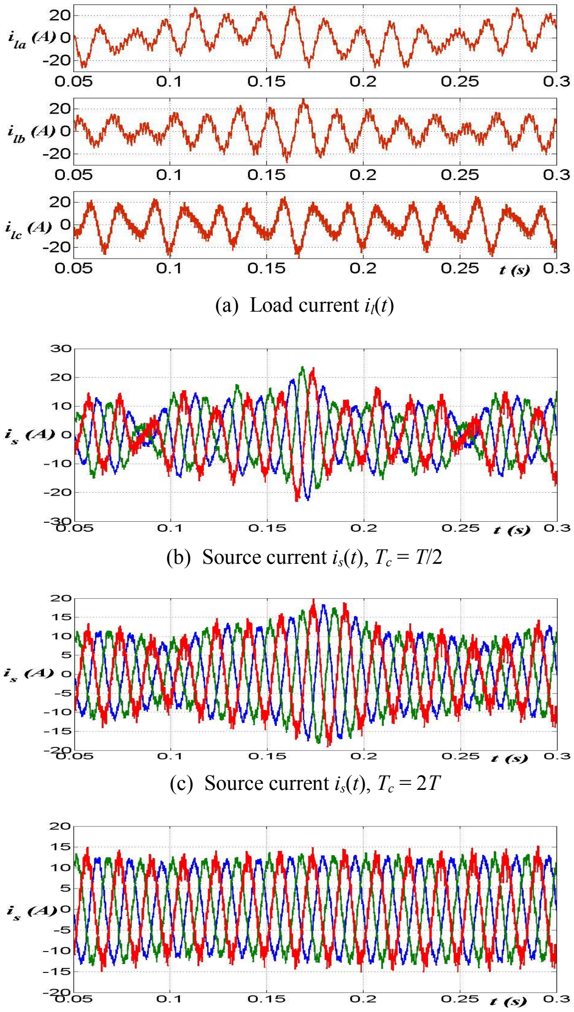
(d) Source current i_s ($T_c = 3T$).

Fig. 5. Simulation of single-phase pulse load compensation.

D. Non-periodic Load

Theoretically, the period T of a non-periodic load is infinite (a period much larger than the fundamental period of the utility voltage). The nonactive components in a load cannot be completely compensated by choosing T_c as $T/2$ or T, or even several multiples of T.

Fig. 6a shows a three-phase non-periodic current. Figs. 6b-6d show the source currents after compensation, with $T_c = T/2$, $2T$, and $10T$, respectively. With $T_c = T/2$ (Fig. 6b), there is still significant nonactive component in i_s with variable peak values and a non-sinusoidal waveform. In Fig. 6c, the variation of the amplitude of i_s is smaller, and in Fig. 6d, i_s is close to a sine wave with less nonactive component. Here, a longer T_c "smoothes" the source current waveforms. Theoretically, i_s could be a pure sine wave if T_c goes to infinity, but in practice, such a T_c cannot be implemented nor is it necessary. If T_c is



(a) Load current $i_l(t)$

(b) Source current $i_s(t)$, $T_c = T/2$

(c) Source current $i_s(t)$, $T_c = 2T$

(d) Source current $i_s(t)$, $T_c = 10T$

Fig. 6. Simulation of non-periodic load compensation.

large enough, increasing T_c further will not typically improve the compensation results significantly. Typically, there is no need to increase T_c to a larger value as the small decrease in THD is often not worth the larger capital costs (higher ratings of the compensator components and therefore higher capital expenses).

V. Discussion

In nonactive power compensation, there are several factors which have significant influence on the choice of compensator type, the power rating of the compensator, the energy storage requirement of the compensator, and the compensation results. Some of these factors are related to the nonactive power theory itself, which include the averaging interval T_c, and the reference voltage $\boldsymbol{v}_p$, while others are practical issues related to the implementation of the compensation system. In this paper, the coupling inductance L_c, the power rating of the compensator, the capacitance of the DC link, and the DC link voltage v_{dc} have been taken into consideration.

A. Averaging Interval T_c

If there are only harmonics in the load current, as in Subsections IV.A and IV.B, T_c does not change the compensation results as long as it is an integral multiple of $T/2$, where T is the fundamental period of the system. Here the nonactive current is completely compensated, and a purely sinusoidal source current is achieved.

However, in other cases, such as in Subsections IV.C and IV.D, T_c has significant influence on the compensation results, and the power and energy storage ratings of the compensator's components. With longer T_c, a better source current will be achieved, but at the cost of higher power rating for the switches and capacitance. There is a tradeoff between better compensation and higher system ratings (i.e., costs). On the other hand, a longer T_c does not necessarily yield a better source current waveform. For a specific system, there is an appropriate T_c with which the compensation can be achieved [12]. For example, in Subsection 4C, the choice of T_c depends on the period of the pulses.

B. Coupling Inductance L_c

The coupling inductance L_c between the power system and the compensator could be the inductance of a step-up transformer or a coupling reactor. It acts as the filter of the compensation current i_c, which has high ripple content due to the compensator's PWM control of the switches.

If L_c is too small, it cannot filter the ripple in the compensation current i_c; and if L_c is too large, the time-constant of the system will be so large that i_c cannot track the reference, which results in inadequate operation of the compensation system.

L_c is inversely proportional to the rms value of load current (I_l) when the ripple in the source current is limited to a specific percentage (5% in this work), i.e.,

$$L_c = K / I_l \tag{20}$$

where K is a constant.

C. DC Link Capacitance C

According to the discussion in Section II, the average power of the compensator $P_c(t)$ over T_c is zero. Energy is neither generated nor consumed by the compensator. Therefore, the energy stored in the capacitor is a constant at rated DC link voltage V_{dc} given by

$$E_c = \frac{1}{2} C V_{dc}^{\ 2}. \tag{21}$$

However, the instantaneous power is not necessarily zero. The compensator generally has a capacitor for energy storage, and this capacitor operates in two modes, i.e., charge and discharge. Different capacitance values are required to fulfill different compensation tasks. The maximum energy variation in the capacitor is the integral of $\boldsymbol{v}(t)\boldsymbol{i}_c(t)$ between time t_{max} when the capacitor goes from discharge to charge, and t_{min} when the capacitor goes from charge to discharge, or vice versa. Thus,

$$\Delta E_c = \int_{t_{min}}^{t_{max}} \boldsymbol{v}(t)^T \boldsymbol{i_c}(t)dt \cdot \quad (22)$$

The energy variation on the DC link capacitor causes the voltage variation, that is

$$E_c + \Delta E_c = \frac{1}{2}C(V_{dc} + \Delta V_{dc})^2 \cdot \quad (23)$$

The DC link voltage variation is a small fraction of V_{dc}, $\Delta V_{dc} = aV_{dc}$, where $a \leq 5\%$. The capacitance requirement is derived from (21) - (23):

$$C = \frac{\Delta E_c}{(a + a^2/2)V_{dc}^2} \cdot \quad (24)$$

For different applications, the energy variation ΔE_c is different, which determines the capacitance rating, for a given DC link voltage variation. For applications other than harmonic load, ΔE_c also changes with T_c [13].

D. DC Link Voltage v_{dc}

The minimum value of the DC link voltage v_{dc} is the peak value of the line-to-line system voltage (V_{ll}) [11]. In practice, a higher v_{dc} is required to allow the PI controller better performance. Here $1.5V_{ll}$ is used in the simulation.

If there is a sudden load change or there is high current variation (di/dt) in the load current, a higher DC link voltage is required so that the compensator has the ability to compensate such a high current variation, which is illustrated in Subsection IV.B.

Besides all the factors discussed above, the switching frequency is also an important factor which needs to be considered. If the load current has very high frequency harmonics, higher switching frequency is required to eliminate the high frequency harmonics.

VI. CONCLUSIONS

A generalized nonactive power theory has been presented in this paper for nonactive power compensation. The instantaneous active current $\boldsymbol{i_a}(t)$, the instantaneous nonactive current $\boldsymbol{i_n}(t)$, the instantaneous active power $p_a(t)$, and the instantaneous nonactive power $p_n(t)$ are defined in a system which does not have any limitations such as the number of phases, whether the voltage and the current are sinusoidal or non-sinusoidal, and whether the load is periodic or non-periodic. By changing the reference voltage $v_p(t)$ and the averaging interval T_c, this theory has the flexibility to define nonactive current and nonactive power in different cases. It is a generalized theory that other nonactive power theories discussed in [2] could be derived from this theory by changing the reference voltage and the averaging interval. The flexibility is illustrated by applying the theory to different cases such as a three-phase periodic system with harmonics, a diode rectifier load, a single-phase pulse load, and a system with non-periodic currents.

This theory is implemented using a shunt compensator. The current that the compensator must provide is calculated based on the generalized nonactive power theory. A control scheme is developed to regulate the DC link voltage of the inverter and to generate the switching signals for the inverter based on the required nonactive current. The compensation system is simulated and different cases are studied. The simulation results show that the theory proposed in this paper is applicable to the nonactive power compensation in three-phase four-wire systems, single-phase systems, load currents with harmonics, and non-periodic load currents. This theory is adapted to different compensation objectives by changing the reference voltage $v_p(t)$ and the averaging interval T_c. The practical issues such as the DC capacitance rating, the coupling inductance, and the DC link voltage are also discussed.

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